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STOCHASTIC MODEL FOR EVALUATING INSURANCE POLICIES IN HEALTH INSURANCE

Markov model of insurance policies with a focus on health insurance is proposed. The formulas for calculating the present value of the payments are derived.
Keywords: voluntary health insurance, a Markov process, the actuarial present value

1. Introduction. Insurance premiums and reserves (especially in the long-term direction of business) are investigated using the present value of cash flows. Insurance case, time and cost of each cash flow is usually unknown in advance. They depend on many random factors [1]. Therefore, our task is to assess their expected values, which are some functional defined on some Markov processes.

2. Principal material. Individual policies, such as health insurance policies, will be represented by means of stochastic processes [2]. To describe the policy should specify:

1. The period of validity of the policy T ;
2. The set of possible states of the insured E ;
3. Stochastic process $\xi(t)$ on T with values of E, reflecting the random changes in the state of the insured.

We assume that T is some interval of $\Re_+ = [0,\infty)$, the set of states E is finite, $E = \{0,1,\dots,n-1\}, \quad n \ge 2$.

Suppose that at any instant of time $t, \quad t \in T$, the policyholder can be in one of a plurality of possible states E. The current state of the insured may be associated with some of cash flows (payments). The problem is to determine the quantitative evaluation of the effect on cash flow state, that is to estimate the probability of the event with the proviso that the individual is in a fixed state.

In what follows we assume that the process $(\xi(t), t \in T)$ is an abrupt regular Markov process [3].

Definition 1. Set $(T, E, (\xi(t), t \in T))$ is called a Markov model of the insurance policy [2].

In this article, we will use the word "process" within the meaning of "insurance", "policy" or "contract." For instance, state i may mean that the insured person is healthy and state j may denote that the insured was ill. Event $\xi(0) = 0$ can be paraphrased as follows: an insurance policy issued in state 0 at time 0.

Transition probabilities of Markov processes from state i at time t to state j at time u will be denoted $p_{ij}(t,u)$ and define the following

$$p_{ij}(t,u) = P[\xi(u) = j | \xi(t) = i] = P[\xi(t) = i, \xi(u) = j] / P[\xi(t) = i], \; u \ge t \ge 0, \; i, j \in E. \qquad (2.1)$$

Transition probabilities satisfy the properties

$$0 \le p_{ij}(t,u) \le 1 \quad i, j \in E; \; \sum_{j \in E} p_{ij}(t,u) = 1 \quad u \ge t \ge 0, \qquad (2.2)$$

$$p_{ij}(t,u) = \sum_{k \in E} p_{ik}(t,s) p_{kj}(s,u), \quad 0 \le t \le s \le u \, . \qquad (2.3)$$

Let us also assume the existence of the following limits

$$\mu_{ij}(t) = \lim_{u \to t} \frac{p_{ij}(u-t)}{u-t} . \qquad (2.4)$$

$i, j \in E, \quad i \neq j$, where $\mu_{ij}(t)$ is transition intensity from state i to state j.

Our goal is to get the transition probabilities for calculating the actuarial values. Transition intensities and transition probabilities are connected by means of differential equations of the Chapman-Kolmogorov [3]:

$$\frac{d}{dt} p_{ij}(z,t) = -\mu_i(t) p_{ij}(z,t) + \sum_{k \neq i}^{n} \mu_{ik}(t) p_{kj}(z,t), \qquad (2.5)$$

For all states i, j and time t, $0 \le z \le t$.

To the system of equations (2.5) to obtain explicit expressions for the transition probabilities assume that the intensity of the transition remains constant while a person is in any state, $\mu_{ij}(t) = \mu_{ij}$ for all t.

Suppose that at any time the insurer r can be in one of the six states:

R – healthy people (those who recovered);

I – group of individuals vulnerable to disease;

L – latent group of persons (such as a result of contact with asymptomatic a or symptomatic infected with the virus received);

S – symptomatically infected;

A – asymptomatically infected;

D –death of the insured.

Let $I = \{0\};. L = \{1\}; \quad S = \{2\}; \quad A = \{3\}; \quad R = \{4\}; \quad D = \{5\}$

Since the set of states is finite, $E = \{0,1,2,3,4\}$, a stochastic process is conveniently illustrated by the so-called graph (Figure 1).

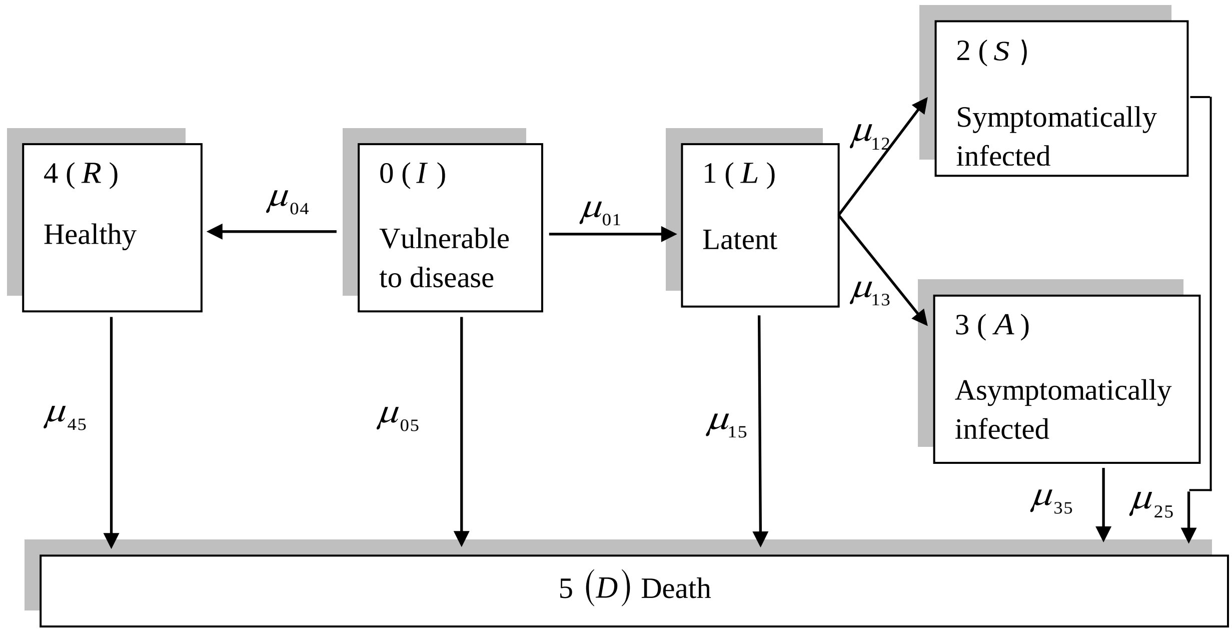

Figure 1

As can be seen from the above graph, the evolution of changes in the state is a homogeneous Markov process with continuous time and finite set of states [4].

Using Figure 1 and the Chapman-Kolmogorov backward system of differential-difference equation (2.5), we obtain the following system of equations:

$$\begin{cases} \frac{d}{dt} p_{00}(t) = -\lambda_0 p_{00}(t) \\ \frac{d}{dt} p_{11}(t) = -\lambda_1 p_{11}(t) \\ \frac{d}{dt} p_{22}(t) = -\mu_{25} p_{22}(t) \\ \frac{d}{dt} p_{33}(t) = -\mu_{35} p_{33}(t) \\ \frac{d}{dt} p_{44}(t) = -\mu_{45} p_{44}(t) \\ \frac{d}{dt} p_{12}(t) = \mu_{12} p_{22}(t) - \lambda_1 p_{12}(t) \\ \frac{d}{dt} p_{13}(t) = \mu_{13} p_{33}(t) - \lambda_1 p_{13}(t) \\ \frac{d}{dt} p_{01}(t) = \mu_{01} p_{11}(t) - \lambda_0 p_{01}(t) \\ \frac{d}{dt} p_{02}(t) = \mu_{01} p_{12}(t) - \lambda_0 p_{02}(t) \\ \frac{d}{dt} p_{03}(t) = \mu_{01} p_{13}(t) - \lambda_0 p_{03}(t) \\ \frac{d}{dt} p_{04}(t) = \mu_{04} p_{44}(t) - \lambda_0 p_{04}(t) \end{cases},$$

where $\lambda_0 = \mu_{01} + \mu_{04} + \mu_{05}$ $\quad \lambda_1 = \mu_{12} + \mu_{13} + \mu_{15}$.

The solution to this system of equations with initial conditions $p_{ii}(0) = 1$, $p_{ij}(0) = 0$ is:

$$p_{00}(t) = e^{-\lambda_0 t}$$
$$p_{11}(t) = e^{-\lambda_1 t}$$
$$p_{22}(t) = e^{-\mu_{25} t}$$
$$p_{33}(t) = e^{-\mu_{35} t}$$
$$p_{44}(t) = e^{-\mu_{45} t}$$
$$p_{12}(t) = \frac{\mu_{12}}{\lambda_1 - \mu_{25}} \left(e^{-\mu_{25} t} - e^{-\lambda_1 t} \right)$$
$$p_{13}(t) = \frac{\mu_{13}}{\lambda_1 - \mu_{35}} \left(e^{-\mu_{35} t} - e^{-\lambda_1 t} \right)$$

$$p_{01}(t)=\frac{\mu_{01}}{\lambda_0-\lambda_1}\left(e^{-\lambda_1 t}-e^{-\lambda_0 t}\right)$$

$$p_{02}(t)=\frac{\mu_{01}\mu_{12}}{(\lambda_0-\lambda_1)(\lambda_0-\mu_{25})(\lambda_1-\mu_{25})}\left(-(\lambda_1-\mu_{25})e^{-\lambda_0 t}+(\lambda_0-\mu_{25})e^{-\lambda_1 t}-(\lambda_0-\lambda_1)e^{-\mu_{25}t}\right)$$

$$p_{03}(t)=\frac{\mu_{01}\mu_{13}}{(\lambda_0-\lambda_1)(\lambda_0-\mu_{35})(\lambda_1-\mu_{35})}\left(-(\lambda_1-\mu_{35})e^{-\lambda_0 t}+(\lambda_0-\mu_{35})e^{-\lambda_1 t}-(\lambda_0-\lambda_1)e^{-\mu_{35}t}\right)$$

$$p_{04}(t)=\frac{\mu_{04}}{\lambda_0-\mu_{45}}\left(e^{-\mu_{45}t}-e^{-\lambda_0 t}\right).$$

We assume that during the disease symptomatically infected individuals, in contrast to asymptomatic infected, require payment of the sum insured provided for in the policy, so consider the insurer in state 2. The probability that the insurer is in state 2 at time t, given that the insurer was able to 0 at time 0 is $p_{02}(t)$.

$$p_{02}(t)=\int_0^t p_{01}(t-u)\mu_{12}du \tag{2.6}$$

$$p_{02}(t)=\frac{\mu_{01}\mu_{12}}{\lambda_0-\lambda_1}\left(\frac{e^{-\lambda_1 t}-1}{\lambda_1}-\frac{e^{-\lambda_0 t}-1}{\lambda_0}\right) \tag{2.7}$$

Calculations within an insurance policy related to the mutual transfer of funds at different points in time. As the value of money changes over time, it is advisable to consider the basic properties of the cash flows.

Change in the value of money is similar to the evolution of risk-free government bonds. Buying one currency for such bonds at time t results in $1+\delta dt$ in the time interval dt in the continuous model, where δ is called the interest rate.

In the described model, we will use the insurance benefits b_t and discounting function υ_t at time t, where υ_t is the discount rate from the date of payment to the date of the contract, and t is the length of time from the date of conclusion of the contract, at the time of symptomatic infection or death.

During t unit cost increases to $e^{\delta t}$ and the discount function for a period of t years are equal $\upsilon_t=e^{-\delta t}$.

Define the function of the present value z_t of the following formula:

$$z_t=b_t\upsilon_t\,. \tag{2.8}$$

Thus z_t denotes the present value of insurance payments at the time of signing the contract. Time of the conclusion of the insurance contract by the time of the insured event (symptomatic infection) for individual age x will be denoted through random variable T^*.

The present value of insurance payments at the time the contract is a random variable z_{T^*}, which in our case is given by the ratio

$$z_{T^*}=b_{T^*}\upsilon_{T^*}\,. \tag{2.9}$$

The first stage of the analysis of health insurance is to determine the values of b_{T^*} and υ_{T^*}.

Insurance against loss occurrence as of n years suggests that the insurance payment will be made only if the policyholder is sick for n years from the date of signing the contract. If at the time of disease people aged x payout occurs sized unit, then

$$b_t = \begin{cases} 1 & t \le n \\ 0 & t > n \end{cases}, \qquad \upsilon_t = \upsilon^t, \quad t \ge 0, \qquad z_{T^*} = \begin{cases} \upsilon^{T^*} & T^* \le n \\ 0 & T^* > n \end{cases}. \tag{2.10}$$

We denote by $Z_t = e^{-\delta t}$ the random variable describing the insurance benefits, when the insured event.

Definition 2. Actuarial present value for persons aged x is the expected value of a random variable current value of Z_t [5].

The actuarial present value (APV) at which the insurance payout sized unit will be denoted through $\overline{A}_{x,n}$, where x is the age of the insured, n – period of insurance.

The actuarial present value of $E[Z_t]$ for the insured aged n years from payment of the size of the unit at the time of the insured event the person (x) denote $\overline{A}_{x,n}$. Since the random variable Z_t is a function of t, then:

$$\overline{A}_{x,n} = E[Z_t] = \int_0^n e^{-\delta t} p'_{02}(t) dt. \tag{2.11}$$

From the points follow the rules, it follows that

$$\overline{B}_{x,n} = D[Z_t] = \overline{A}^2_{x,n} - \left(\overline{A}_{x,n}\right)^2. \tag{2.12}$$

Where $\overline{A}^2_{x,n}$ – the actuarial present value of insurance for n years from the payment of unit size with the intensity of accrual of interest δ.

$\overline{B}_{x,n}$ is a weighted average of the squared deviations of the actual financial results of the medium.

Qualitative analysis also involves identifying risks, identify sources and causes of. According to this concept corresponds to the standard deviation, which is defined by the formula

$$\overline{R}_{x,n} = \sqrt{D[Z_t]}. \tag{2.13}$$

The economic content of the standard deviation in terms of the theory of risk characterization is the maximum possible fluctuations of the investigated parameter from its average expected value.

The greater the variance and standard deviation, the riskier the administrative decision.

Using formula (2.11), (2.12) and (2.13), we obtain the following expressions:

$$\overline{A}_{x,n} = \frac{\mu_{01}\mu_{12}}{(\lambda_0 - \lambda_1)}\left(\frac{\left(e^{-(\delta+\lambda_1)n} - 1\right)}{(\lambda_1 + \delta)} - \frac{\left(e^{-(\delta+\lambda_0)n} - 1\right)}{(\lambda_0 + \delta)}\right)$$

$$\overline{B}_{x,n} = \frac{\mu_{01}\mu_{12}}{(\lambda_0 - \lambda_1)}\left(\frac{\left(e^{-(2\delta+\lambda_1)n} - 1\right)}{(\lambda_1 + 2\delta)} - \frac{\left(e^{-(2\delta+\lambda_0)n} - 1\right)}{(\lambda_0 + 2\delta)}\right) - \frac{\mu^2_{01}\mu^2_{12}}{(\lambda_0 - \lambda_1)^2}\left(\frac{\left(e^{-(\delta+\lambda_1)n} - 1\right)}{(\lambda_1 + \delta)} - \frac{\left(e^{-(\delta+\lambda_0)n} - 1\right)}{(\lambda_0 + \delta)}\right)^2$$

$$\overline{R}_{x,n} = \sqrt{\overline{B}_{x,n}} ..$$

Consider also cases where δ is random variable, which is responsible for the interest rate:

1. δ is a continuous random variable;
2. δ is a discrete random variable.

Stop at each case in more detail.

1. Let δ be continuous random variable with distribution function $F(x)$, $F(x) = P\{\delta < x\}$. Then the actuarial present value for the insured aged n years from payment of the size of the unit at the

time of the insured event can be calculated as follows:

$$\overline{A}_n = E[Z_t] = \int_0^n p'_{02}(t)\left(\int_0^\infty e^{-\delta t} dF(\delta)\right)dt = \int_0^n p'_{02}(t)\int_0^\infty e^{-\delta t} f(\delta)d\delta\,, \qquad (2.14)$$

where $f(\delta)$ is the density function of the random variable δ.

2. δ is the random variable that takes the value of $\delta_1, \delta_2, \ldots, \delta_k$ with probabilities $p_1, p_2, \ldots, p_k$, respectively. If $p_l = P\{\delta = x_l\}$, then $\sum_{l=1}^{k} p_l = 1$. Then the actuarial present value is calculated as follows:

$$\overline{A}_{x,n} = E[Z_t] = \sum_k p_k \int_0^n e^{-\delta_k t} p'_{02}(t)dt\,. \qquad (2.15)$$

3. Conclusion. We show how the Markov process can be used as part of the actuarial modeling of certain types of long-term insurance. An attempt is made to assess transition probabilities, basic insurance and the cost of medical services.

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Tibbi sığorta polislərinin qiymətləndirilməsi üçün stoxastik model

Tibbi sığorta timsalında sığorta polislərinin Markov modeli qurulmuşdur. Ödəniş dəyərinin hesablanması üçün düsturlar alınmışdır.

Açar sözlər: könüllü tibbi sığorta, Markov prosesi, cari aktual dəyər

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Стохастическая модель для оценивания страховых полисов в здравоохранении

Предложена Марковская модель страховых полисов с акцентом на медицинское страхование. Выведены формулы для расчета приведенной стоимости выплат.

Ключевые слова: добровольное медицинское страхование, процесс Маркова, актуарная текущая стоимость

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