Informatics and Control Problems 39 Issue 1 (2019) 26-33
journal homepage: www.icp.az

Invariant control and filtering results for wide band noises

A.E. Bashirov*

[a] *Eastern Mediterranean University, Famagusta, North Cyprus*
[b] *Institute of Control Systems of Azerbaijan National Academy of Sciences, Baku, Azerbaijan*

A R T I C L E   I N F O

Article history:
Received 07.05.2019
Received in revised form 03.06.2019
Accepted 10.06.2019
Available online 12.06.2019

Keywords:
White noise
Wide band noise
Filtering
Stochastic control

A B S T R A C T

The most popular noise model employed in studying stochastic systems is a white noise model. At the same time, the real noises are rarely white. They are rarely colored as well. Mostly, they are wide band. This pushes to develop mathematical methods of studying stochastic systems corrupted by wide band noises. Engineers detect wide band noises by their autocovariance functions, which do not allow to model them uniquely. Therefore, it becomes important to develop mathematical methods which are independent of wide band noises having the same autocovariance function. Such results are called invariant results. In this paper, we review some invariant estimation and control results for wide band noise driven stochastic systems.

1. Introduction

Noise is an indestructible attribute of nature. Therefore, the scientists have been studying methods of working under noises. The most popular noise model employed by system scientists is a white noise. Somewhat improved noise model is a colored noise. But, the real noises are rarely white and colored. They are mostly wide band.

To the best of the author's knowledge, a first record about wide band noises appears in the book [1] by Fleming and Rishel, although earlier discussions in the engineering literature are not excluded as well. Later, wide band noises were prompted in [2] by Kushner and Runggaldier. At present, there are two approaches to wide band noises. A method by approximations is developed in a series of works [2-11]. A different method by integral representation was suggested in [12-16] which leads to modeling wide band noises as a distributed delay of white noises [17, 18].

White noise driven systems have a strong mathematical basis, provided by Ito's stochastic calculus [19, 20]. Moreover, a replacement of wide band noises by white noises produces results which are more or less acceptable in some applied areas. These factors prompted the acceptance of white noises as a popular noise model. At the same time, technology needs more and more delicate study of stochastic systems. In this way, a modification of the results for white noise driven systems to wide band noise driven systems becomes important. In this paper, we are aimed to present some already proved results in the areas of estimation and stochastic control of linear and nonlinear stochastic systems along the second of the above mentioned approaches.

We prefer to write the arguments of functions in subscripts, for example, f_t instead of $f(t)$. This is done to make shorter big expressions. The other general notations are as follows. $\mathbb{R}^n$ is the n-dimensional Euclidean space. As always, $\mathbb{R}^1 = \mathbb{R}$. $\mathbb{R}^{n\times m}$ is a space of $(n\times m)$-matrices. The transpose of A is denoted by A^*. We write $A\geq 0$ for $A\in\mathbb{R}^{n\times n}$ if $A=A^*$ and $\langle x, Ax\rangle \geq 0$ for all

* *E-mail addresses:* agamirza.bashirov@emu.edu.tr (A.E. Bashirov)

www.icp.az/2019/1-03.pdf

$x \in \mathbb{R}^n$. We also write $A > 0$ if $A = A^*$ and $\langle x, Ax \rangle > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$. tr$P$ means the trace of the matrix P. The expectation and conditional expectation of the random variable ξ are denoted by $\mathbf{E}\xi$ and $\mathbf{E}(\xi|\cdot)$, respectively. $\mathrm{cov}(\xi, \eta)$ denotes the covariance of ξ and η. We let $\mathrm{cov}\xi = \mathrm{cov}(\xi, \xi)$. Some other notation will be introduced in the text.

2. Motivation

For a given square integrable random process φ, denote

$$\Lambda_{t,\theta} = \mathrm{cov}(\varphi_{t+\theta}, \varphi_t), \quad t, \theta \geq 0. \tag{1}$$

If $\Lambda_{t,\theta} \neq 0$ for $0 \leq \theta < \varepsilon$ and $\Lambda_{t,\theta} = 0$ for $\theta \geq \varepsilon$, where $\varepsilon > 0$, then φ is said to be a wide band noise, which is stationary (in the wide sense) if, additionally, $\mathbf{E}\varphi_t = 0$ and Λ depends just on its second argument. In the case when $\Lambda_{t,\theta} = \delta_\theta$, where δ is the Dirac's delta function, φ becomes a white noise. Therefore, a white noise is an ideal limit case of wide band noises when $\varepsilon \to 0^+$ so that $\Lambda_{t,0} \to \infty$.

Although the real noises are wide band, a replacement of them by white noises produces results which are more or less acceptable in applications. To explain this phenomena, consider a standard Wiener process v (for simplicity, one-dimensional). Its derivative does not exist in the ordinary sense. We can look to its generalized derivative v', which is called a white noise. Therefore, informally

$$v_t' = \lim_{\varepsilon \to 0} \frac{v_{t+\varepsilon} - v_t}{\varepsilon}. \tag{2}$$

For random processes, this limit should be treated as the left limit. Then

$$v_t' = \lim_{\varepsilon \to 0^+} \frac{v_{t-\varepsilon} - v_t}{-\varepsilon} = \lim_{\varepsilon \to 0^+} \int_{t-\varepsilon}^{t} \frac{1}{\varepsilon} dv_s. \tag{3}$$

Denote

$$\phi_t = \int_{t-\varepsilon}^{t} \frac{1}{\varepsilon} dv_s. \tag{4}$$

Calculations show that

$$\lambda_{t,\theta} = \mathrm{cov}(\phi_{t+\theta}, \phi_t) = \mathbf{E}(\phi_{t+\theta}\phi_t) = \tfrac{\varepsilon - \theta}{\varepsilon^2} \neq 0 \tag{5}$$

if $0 \leq \theta < \varepsilon$ and $\mathrm{cov}(\phi_{t+\theta}, \phi_t) = 0$ if $\theta \geq \varepsilon$. This means that ϕ is a wide band noise and λ is its autocovariance function.

This explaines the preceding phenomena as follows. In the cases when ε in (4) is sufficiently small, a replacement of the wide band noise ϕ by white noise v' produces mathematical results which reflect the reality with more or less acceptable accuracy. But for more adequate mathematical results, wide band noises with representation in (4) should be considered.

More generally, the representation in (4) can be written as

$$\varphi_t = \int_{\max(0, t-\varepsilon)}^{t} \Phi_{t,s-t} dw_s, \qquad t \geq 0, \tag{6}$$

where $\varepsilon > 0$, w is a vector-valued standard Wiener process, and Φ is a matrix-valued non-random function on $[0, \infty) \times [-\varepsilon, 0]$. Then

$$\Lambda_{t,\theta} = \mathrm{cov}(\varphi_{t+\theta}, \varphi_t) = \int_{\max(0,t+\theta-\varepsilon)}^{t} \Phi_{t+\theta,s-t-\theta}\Phi^*_{t,s-t}ds \neq 0 \qquad (7)$$

if $0 \le \theta < \varepsilon$ and $\Lambda_{t,\theta} = 0$ if $\theta \ge \varepsilon$. Therefore, φ is a wide band noise and (6) is its integral representation. The function Φ in (6) is called a relaxing function of φ. The wide band noise φ is stationary on $[\varepsilon, \infty)$ if Φ is independent on its first argument. In this case,

$$\Lambda_{t,\theta} = \int_{\max(-t,\theta-\varepsilon)}^{0} \Phi_{s-\theta}\Phi^*_s ds, \qquad (8)$$

implying $\Lambda_{t,\theta} \equiv \Lambda_\theta$ if $t \ge \varepsilon$.

In applied problems, engineers detect wide band noises by their autocovariance functions. If Φ is a solution of (8) under given Λ, then $-\Phi$ is also a solution. Generally, in [21, 22] it is shown that for a given positive definite function Λ, there are infinitely many relaxing functions Φ producing infinitely many wide band noises with the same autocovariance function Λ. Therefore, the integral form requires making an appropriate selection among all infinitely many relaxing functions. In this regard, the results which are independent on the infinitely many relaxing functions, but dependent on the unique autocovariance function become very important. Such results are called invariant results. The invariance requires to specify a class of relaxing functions Φ which do not change the result. Below, we will use the following invariances:

- L_2-*invariance*. Let $L_2 = L_2(-\varepsilon, 0; \mathbb{R}^{n\times k})$ be the space of square integrable with respect to the Lebesgue measure $\mathbb{R}^{n\times k}$-valued functions on $[-\varepsilon, 0]$. If a result remains the same for all wide band noises with the same autocovariance function Λ so that the respective relaxing functions Φ are continuous in *t* and take values in L_2, then it will be called L_2-invariant.
- $W_0^{1,2}$-*invariance*. Let $W_0^{1,2} = W_0^{1,2}(-\varepsilon, 0; \mathbb{R}^{n\times k})$ be the space of $\mathbb{R}^{n\times k}$-valued functions on $[-\varepsilon, 0]$ with the representation $f_\theta = \int_{-\varepsilon}^{\theta} g_s\, ds$ for some $g \in L_2(-\varepsilon, 0; \mathbb{R}^{n\times k})$. If a result remains the same for all wide band noises with the same autocovariance function Λ so that the respective relaxing functions Φ are continuous in *t* and take values in $W_0^{1,2}$, then it will be called $W_0^{1,2}$-invariant.

This paper is aimed to present some L_2- and $W_0^{1,2}$-invariant estimation and control results for wide band noise driven systems.

3. Linear filtering

Generally speaking, given a pair (y, z) of random processes, a filtering problem consists of finding a finite number of equations for the conditional expectation

$$\hat{y}_t = \mathbf{E}(y_t | z_s, 0 \le s \le t). \qquad (9)$$

A most popular and widely used filtering result is the so-called Kalman filtering [23, 24], which is stated for the white noise driven linear signal-observation system

$$\begin{cases} dy_t = Ay_t dt + \Phi dw_t, & y_0 = \xi, \quad t > 0, \\ dz_t = Cy_t dt + dv_t, & z_0 = 0, \quad t > 0, \end{cases} \qquad (10)$$

where $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{m\times n}$, $\Phi \in \mathbb{R}^{n\times k}$, w and v are independent (for simplicity) standard k- and m-dimensional Wiener processes, ξ is a Gaussian random variable which is independent on (w, v) and $\mathbf{E}\xi = 0$. Kalman filtering result states that

$$d\hat{y}_t = A\hat{y}_t + P_t C^*(dz_t - C\hat{y}_t dt), \qquad \hat{y}_0 = 0, \qquad t > 0, \qquad (11)$$

where P is the solution of the Riccati equation

$$P_t' = AP_t + P_tA^* + \Phi\Phi^* - P_tC^*CP_t, \qquad P_0 = \operatorname{cov}\xi, \qquad t > 0, \tag{12}$$

with the mean square error $e_t = \mathbf{E}\,\|\hat{y}_t - y_t\|^2 = \operatorname{tr}P_t$.

Modifications of the Kalman filter are proved in [25] for a wide band noise driven signal system and in [26] for a wide band noise driven observation system as stated below in Subsections (9) and (10) in simple forms. More general result, combining these two results and considering correlation between the disturbing wide band noises, is proved in [27]. These results pushes to consider filtering problems with pointwise delays of white noises as well [28-30].

3.1. Wide band noise driven signal. Consider the preceding system with the wide band noise driven signal

$$\begin{cases} y_t' = Ay_t + \varphi_t, & y_0 = \xi, \quad t > 0, \\ dz_t = Cy_t\,dt + dv_t, & z_0 = 0, \quad t > 0, \end{cases} \tag{13}$$

where additionally we assume that φ is a wide band noise with the autocovariance function Λ and independent on (ξ, v). Then the L_2-invariant best estimate $\hat{y}$ is a unique solution of the equation

$$d\hat{y}_t = \big(A\hat{y}_t + \psi_{t,0}\big)dt + P_tC^*(dz_t - C\hat{y}_t dt), \qquad \hat{y}_0 = 0, \qquad t > 0, \tag{14}$$

where ψ is a unique solution of the equation

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta}\right)\psi_{t,\theta}dt = M_{t,\theta}C^*(dz_t - C\hat{y}_t dt), \\ \psi_{0,\theta} = \psi_{t,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, t > 0. \end{cases} \tag{15}$$

Here, P is a unique solution of the Riccati equation

$$P_t' = AP_t + P_tA^* + M_{t,0} + M_{t,0}^* - P_tC^*CP_t, \qquad P_0 = \operatorname{cov}\xi, \qquad t > 0, \tag{16}$$

where M is uniquely determined as a solution of the equation

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta}\right)M_{t,\theta} = M_{t,\theta}A^* + \Lambda_{t,-\theta} - N_{t,\theta,0} - M_{t,\theta}C^*CP_t, \\ M_{0,\theta} = M_{t,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, t > 0. \end{cases} \tag{17}$$

and N of the equation

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta} + \dfrac{\partial}{\partial \tau}\right)N_{t,\theta,\tau} = M_{t,\theta}C^*CM_{t,\tau}^* \\ N_{0,\theta,\tau} = N_{t,-\varepsilon,\tau} = N_{t,\theta,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, -\varepsilon \le \tau \le 0, t > 0. \end{cases} \tag{18}$$

The mean square error is equal to $e_t = \mathbf{E}\|x_t - x_t\|^2 = \operatorname{tr}P_t$.

3.2. Wide band noise driven observations. Consider the preceding system with the wide band noise driven observation system in the form

$$\begin{cases} dy_t = Ay_t\,dt + \Phi dw_t, & y_0 = \xi, \quad t > 0, \\ dz_t = (Cy_t + \varphi_t)dt + dv_t, & z_0 = 0, \quad t > 0, \end{cases} \tag{19}$$

where besides the above mentioned conditions we assume that φ is a wide band noise with the autocovariance function Λ and independent on (ξ, w, v). Then the $W_0^{1,2}$-invariant best estimate $\hat{y}$ is a unique solution of the equation

$$d\hat{y}_t = A\hat{y}_t\,dt + (P_tC^* + M^*_{t,0})\big(dz_t - C\hat{y}_t dt - \psi_{t,0}dt\big), \qquad \hat{y}_0 = 0, \qquad t > 0, \tag{20}$$

where ψ is a wide band noise obtained as a solution of

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta}\right)\psi_{t,\theta}dt = (M_{t,\theta}C^* + \Lambda_{t,-\theta} - N_{t,\theta,0})\big(dz_t - Cx\hat{y}_t dt - \psi_{t,0}dt\big), \\ \psi_{0,\theta} = \psi_{t,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, t > 0. \end{cases} \tag{21}$$

Here, P is a solution of the Riccati equation

$$P'_t = AP_t + P_tA^* + \Phi\Phi^* + \big(P_tC^* + M^*_{t,0}\big)\big(CP_t + M_{t,0}\big), \qquad P_0 = \mathrm{cov}\xi, \qquad t > 0, \tag{22}$$

where M is uniquely determined as a solution of the equation

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta}\right)M_{t,\theta} = M_{t,\theta}A^* - (M_{t,\theta}C^* + \Lambda_{t,-\theta} - N_{t,\theta,0})(CP_t + M_{t,0}), \\ M_{0,\theta} = M_{t,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, t > 0, \end{cases} \tag{23}$$

and N of the equation

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial \theta} + \dfrac{\partial}{\partial \tau}\right)N_{t,\theta,\tau} = \big(M_{t,\theta}C^* + \Lambda_{t,-\theta} - N_{t,\theta,0}\big)\big(CM^*_{t,\tau} + \Lambda^*_{t,-\tau} - N^*_{t,\tau,0}\big), \\ N_{0,\theta,\tau} = N_{t,-\varepsilon,\tau} = N_{t,\theta,-\varepsilon} = 0, -\varepsilon \le \theta \le 0, -\varepsilon \le \tau \le 0, t > 0. \end{cases} \tag{24}$$

Additionally, $e_t = \mathbf{E}\|x_t - x_t\|^2 = \mathrm{tr}P_t$.

4. Stochastic control

A most popular and completely solved stochastic optimal control problem is a so-called linear quadratic Gaussian (LQG) problem. In a simple form, it is stated as a minimization of the functional

$$J(u) = \mathbf{E}\left(\langle x_T, Hx_T\rangle + \int_0^T (\langle x_t, Fx_t\rangle + \langle u_tGu_t\rangle)dt\right) \tag{25}$$

subjected to the partially observable linear system

$$\begin{cases} dy_t = (Ay_t + Bu_t)dt + \Phi dw_t, \ x_0 = \xi, \ 0 < t \le T, \\ dz_t = Cy_tdt + dv_t, \qquad\qquad z_0 = 0, \ 0 < t \le T, \end{cases} \tag{26}$$

over the controls u in the linear feedback form. Here, in addition to the preceding conditions, $B \in \mathbb{R}^{n\times l}$, $H, F \in \mathbb{R}^{n\times n}$ with $H \ge 0$ and $F \ge 0$, $G \in \mathbb{R}^{l\times l}$ with $G > 0$. There exists an optimal control u^* in this LQG problem together with the optimal state y^* and corresponding observation process z^*such that

$$u^*_t = -G^{-1}B^*Q_t\hat{y}^*_t, \qquad 0 \le t \le T. \tag{27}$$

The best estimate $\hat{y}^*_t = \mathbf{E}(y^*_t | z^*_s, 0 \le s \le t)$ is a unique solution of the equation

$$d\hat{y}^*_t = (A\hat{y}^*_t + Bu^*_t)dt + P_tC^*(dz^*_t - C\hat{y}^*_t dt), \qquad \hat{y}^*_0 = 0, \qquad 0 < t \le T. \tag{28}$$

Here Q is a unique solution of the backward Riccati equation

$$Q'_t + A^*Q_t + Q_tA + F - Q_tBG^{-1}B^*Q_t, \qquad Q_T = H, \qquad 0 \le t < T, \tag{29}$$

and P of the forward Riccati equation

$$P'_t = AP_t + P_tA^* + \Phi\Phi^* - P_tC^*CP_t, \qquad P_0 = \mathrm{cov}\xi, \qquad 0 < t \le T. \tag{30}$$

Modification of this result to the case of wide band noise driven systems is proved in [31] and presented below in a simple case.

4.1. Wide band noise driven state. Consider LQG problem of minimizing the preceding functional subject to the state-observation system

$$\begin{cases} y_t' = Ay_t + Bu_t + \varphi_t, & y_0 = \xi, \quad 0 < t \le T, \\ dz_t = Cy_t dt + dv_t, & z_0 = 0, \quad 0 < t \le T. \end{cases} \tag{31}$$

There exists a unique L_2-invariant optimal control u^* in this LQG problem determined by

$$u_t^* = -G^{-1}B^*Q_t\hat{y}_t^* - G^{-1}B^*\int_t^{\min(T,t+\varepsilon)} \mathcal{Y}_{s,t}^* \, Q_s\psi_{t,t-s}ds, \qquad 0 \le t \le T, \tag{32}$$

with the optimal state and observations y^* and z^*. The best estimate $\hat{y}_t^* = \mathbf{E}(y_t^*|z_s^*, 0 \le s \le t)$ is a unique solution of the equation

$$d\hat{y}_t^* = \left(A\hat{y}_t^* + \psi_{t,0} + Bu_t^*\right)dt + P_tC^*(dz_t^* - C\hat{y}_t^*\,dt), \qquad \hat{y}_0^* = 0, \qquad 0 < t \le T. \tag{33}$$

Here ψ, (P, M, N), and Q are unique solutions of (15), (16)-(18), and (29), respectively, $\mathcal{Y}$ is a bounded perturbation of the semigroup e^{At} by $-BG^{-1}B^*Q_t$.

4.2. Wide band noise driven observations. Consider the preceding LQG-problem with wide band noise driven observation system in the form

$$\begin{cases} dy_t = (Ay_t + Bu_t)\,dt + \Phi dw_t, & y_0 = \xi, \quad 0 < t \le T, \\ dz_t = (Cy_t + \varphi_t)dt + dv_t, & z_0 = 0, \quad 0 < t \le T. \end{cases} \tag{34}$$

There exists a unique $W_0^{1,2}$-invariant optimal control u^* in this LQG problem determined by

$$u_t^* = -G^{-1}B^*Q_t\hat{y}_t^*, \qquad 0 \le t \le T, \tag{35}$$

with the optimal state and observation processes y^* and z^*. The best estimate $\hat{y}_t^* = \mathbf{E}(y_t^*|z_s^*, 0 \le s \le t)$ is a unique solution of the equation

$$d\hat{y}_t^* = (A\hat{y}_t^* + Bu_t^*)dt + (P_tC^* + M_{t,0}^*)\left(dz_t^* - C\hat{y}_t^*dt - \psi_{t,0}dt\right), \hat{y}_0^* = 0, 0 < t \le T. \tag{36}$$

Here ψ, (P, M, N), and Q are unique solutions of (21), (22)-(24), and (29), respectively.

4.3. Stochastic maximum principle. Consider a wide band noise $[\varphi(y)]_t$ depending on the parameter $y \in \mathbb{R}^n$. Its autocovariance function $[\Lambda(y)]_{t,\theta}$ depends on y as well. If it has a representation of the form (6) for some Wiener process w, then its relaxing function $[\Phi(y)]_{t,\theta}$ also depends on y. Define

$$[\Psi(y)]_{t,\theta} = \begin{cases} [\Phi(y)]_{t-\theta,\theta} & \text{if } t - \theta \le T, \\ 0 & \text{if } t - \theta > T. \end{cases} \tag{37}$$

We can consider the problem of minimizing the nonlinear functional

$$J(u) = \mathbf{E}\left(h(y_T) + \int_0^T g(t, y_t, u_t)dt\right) \tag{38}$$

over the system

$$y_t' = f(t, y_t, u_t) + [\varphi(y_t)]_t, \qquad y_0 = \xi, \qquad 0 < t \le T, \tag{39}$$

assuming the following standard conditions for the functions $f: [0, T] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $\Psi: [0, T] \times \mathbb{R}^n \to L_2(-\varepsilon, 0; \mathbb{R}^{n\times k})$, $h: \mathbb{R}^n \to \mathbb{R}$, and $g: [0, T] \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$:

- f, Ψ, h, and g are measurable and differentiable in y,
- $\partial f/\partial y$ and $\partial\Psi/\partial y$ are bounded,

- f, Ψ, $\partial g/\partial y$, and $\partial h/\partial y$ are bounded by $c(1 + \|y\| + \|u\|)$,
- h and g are bounded by $c(1 + \|y\|^2 + \|u\|^2)$

where c is a constant. Let $U \subset \mathbb{R}^m$ be nonempty and measurable. Choose the set of admissible controls to be all square integrable and $\mathcal{F}_t$-adapted random processes on $[0,T]$ with values in U and let ξ be $\mathcal{F}_0$-measurable, where $\{\mathcal{F}_t\}$ is a natural filtration generated by w. Define the Hamiltonian by

$$\mathcal{H}(t,y,u,p) = -g(t,y,u) + \langle p, f(t,y,u)\rangle \quad (40)$$

and consider the backward stochastic differential equation

$$dp_t = -\mathcal{H}_y'(t,y_t,u_t,p_t)dt + q_t dw_t, \qquad p_T = -h_y'(y_T), \qquad 0 \le t < T. \quad (41)$$

Under the above conditions and notation, the following L_2-invariant maximum principle holds: if u^* is an optimal control in this problem together with the optimal state y^* and the respective solution (p^*, q^*) of the backward equation, then

$$\max_{v\in U}\mathcal{H}(t,y_t^*,v,p_t^*) = \mathcal{H}(t,y_t^*,u_t^*,p_t^*). \quad (42)$$

5. Conclusion.

Some optimal control and filtering results are presented in this article. They are proved in different papers for wide band noise driven systems. An essential feature of these results is that they are invariant and, therefore, need not to know relaxing functions. In this regard, an application of these results requires determination of the respective autocovariance function (by time series analysis, for example) and numerical solution of the presented equations.

References

[1] W.M. Fleming, R.W. Rishel, Deterministic and Stochastic Optimal Control, Springer-Verlag, 1975.
[2] H.J. Kushner, W.J. Runggaldier, Filtering and control for wide bandwith noise driven systems, IEEE Trans. Automat. Control. 32-AC (1987) pp.123-133.
[3] M. Al-Ansary, H. Khalil, On the interplay of singular perturbations and wide-band stochastic fluctuations, SIAM J. Contr. 24 (1986) 83-98.
[4] G.B. Blankenship, G.C. Papanicolaou, Stability and control of stochastic systems with wide band noise disturbances, SIAM J. Appl. Math. 34 (1978) 437-476.
[5] H.J. Kushner, Jump diffusion approximations for ordinary differential equations with wideband random right sides, SIAM J. Contr. Optim. 17 (1979) 729-744.
[6] H.J. Kushner, Diffusion approximations to output processes of nonlinear systems with wide-band inputs, and application, IEEE Trans. Inform. Theory. IT-26 (1980) 715-725.
[7] H.J. Kushner, W.J. Runggaldier, Nearly optimal state feedback controls for stochastic systems with wideband noise disturbances, SIAM J. Contr. Optim. 25 No.2 (1987) 298-315.
[8] H.J. Kushner, K.M. Ramachandran, Nearly optimal singular controls for sideband noise driven systems, SIAM J. Contr. and Optim. 26 No.3 (1988) 569-591.
[9] H.J. Kushner, Weak convergence methods and singularly perturbed control and filtering problems, Birkhäuser, Boston, 1990.
[10] R.S. Liptser, W.J. Runggaldier, M. Taksar, Diffusion approximation and optimal stochastic control. Theory of Probability and Applications. 44 (2000) 69-698.
[11] H.J. Kushner, Modelling and approximations for stochastic systems with state-dependent singular controls and wide-band noise, SIAM J. Contr. Optim. 52 No.1 (2014) 311-338.
[12] A.E. Bashirov, Optimal control of partially observed systems with arbitrary dependent noises, Stochastics, 17 (1986) 163-205.
[13] A.E. Bashirov, On linear filtering under dependent wide band noises, Stochastics. 23 (1988) 413-437.
[14] A.E. Bashirov, Control and filtering for wide band noise driven linear systems, Journal of Guidance, Control, and Dynamics. 16 No.5 (1993) 983-985.
[15] A.E. Bashirov, H. Etikan, N. Şemi, Filtering, smoothing and prediction for wide band noise driven systems, J. Franklin Inst. 334B(4) (1997) 667-683.

[16] A.E. Bashirov, L.V. Eppelbaum, L.R. Mishne, Improving Eötvös corrections by wide band noise Kalman filtering, Geophys. J. Int. 108 No.1 (1992) 193-197.
[17] A.E. Bashirov, Z. Mazhar, H. Etikan, and S. Ertürk, Delay structure of wide band noises with application to filtering problems, Optimal Control Appl. Methods. 34 No.1 (2013) 69-79.
[18] A.E. Bashirov, Z. Mazhar, H. Etikan, S. Ertürk, Boundary value problems arising in Kalman filtering, Boundary Value Problems, 2008. https://doi.org/10.1155/2008/279410
[19] K. Ito, Stochastic integral, Proceedings of the Imperial Academy, Tokyo. 20 (1944) 519-524.
[20] K. Ito, On stochastic differential equations, Memoirs of the American Mathematical Siciety. 4 (1951) 645-668.
[21] A.E. Bashirov, S. Uğural, Analyzing wide band noise processes with application to control and filtering, IEEE Trans. Automat. Control. 47-AC(2) (2002) 323-327.
[22] A.E. Bashirov, S. Uğural, Representation of systems disturbed by wide band noises, Appl. Math. Lett. 15 No.5 (2002) 607-613.
[23] R.E. Kalman, A new approach to linear filtering and prediction problem, Trans. ASME (J. Basic Eng.) Ser. D, 82 (1960) 35-45.
[24] R.E. Kalman, R.S. Bucy, New results in linear filtering and prediction theory, Trans. ASME (J. Basic Eng.) Ser. D. 83 (1961) 95-108.
[25] A.E. Bashirov, K. Abuassba, Invariant linear filtering and control results for wide band noise driven signal systems, TWMS J. Appl. Eng. Math. 8 No.1 (2018) 71-82.
[26] A.E. Bashirov, Linear filtering for wide band noise driven observation systems, Circuits Syst. Signal Process. 36 No.3 (2017) 1247-1263.
[27] A.E. Bashirov, K. Abuassba, Invariant Kalman filter for correlated wide band noises, Asian Journal of Control. 22 No.1 (2020) 1-9. https://doi.org/10.1002/asjc.1949
[28] A.E. Bashirov, Filtering for linear systems with shifted noises, Internat. J. Contr. 78 No.7 (2005) 521-529.
[29] A.E. Bashirov, Z. Mazhar, S. Ertürk, Kalman type filter for systems with delay in observation noise, Appl. Comput. Math. 12 No.3 (2013) 325-338.
[30] A.E. Bashirov, Z. Mazhar, On asymptotical behaviour of solution of Riccati equation arising in linear filtering with shifted noises, Mathematical Methods in Engineering, Proceedings of the International Symposium on Mathematical Methods in Engineering, Ankara, April 27--29, 2006, K.Taş, J.A.Tenriero Machado and D.Baleanu (eds.), Springer-Verlag, Dordrecht. (2007) 141-149.
[31] A.E. Bashirov, Partially observable linear systems under dependent noises, Systems & Control: Foundations & Applications, Birkhäuser, Basel, 2003.
[32] A.E. Bashirov, Stochastic maximum principle in the Pontryagin's form for wide band noise driven systems, Internat. J. Control. 88 No.3 (2015) 461-468.