Informatics and Control Problems 39 Issue 1 (2019) 56-63
journal homepage: www.icp.az

Methods for finding suboptimistic and subpessimistic solutions to interval part of the integer programming problem

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ARTICLE INFO

Article history:
Received 15.10.2018
Received in revised form 12.12.2018
Accepted 22.05.2019
Available online 12.06.2019

Keywords:
Permissible
Optimistic
Pessimistic
Lower and upper bounds
Computational experiments
Errors

ABSTRACT

A mixed-integer programming problem with interval source data is considered. The concepts of admissible, optimistic, pessimistic, suboptimistic and subpessimistic solutions for the problem under investigation are introduced. Based on one economic interpretation of the problem, methods for finding suboptimistic and subpessimistic solutions are proposed. The programs of the developed method are compiled and a number of computational experiments are carried out. The mean estimates of the errors of these solutions in optimistic and pessimistic solutions are found. As a result of experiments, the effectiveness of the developed methods is confirmed once again.

1. Introduction

The following interval mixed-integer programming problem is considered:

$$\sum_{j=1}^{n}\left[\underline{c}_j,\overline{c}_j\right]x_j+\sum_{j=n+1}^{N}\left[\underline{c}_j,\overline{c}_j\right]x_j\to\max \tag{1}$$

$$\sum_{j=1}^{n}\left[\underline{a}_{ij},\overline{a}_{ij}\right]x_j+\sum_{j=n+1}^{N}\left[\underline{a}_{ij},\overline{a}_{ij}\right]x_j\le\left[\underline{b}_i,\overline{b}_i\right],\quad (i=\overline{1,m}) \tag{2}$$

$$0\le x_j\le d_j,\ \left(j=\overline{1,N}\right), \tag{3}$$

$$x_j,\text{ integers},\ \left(j=\overline{1,n}\right),\ (n\le N) \tag{4}$$

Here it is assumed that the coefficients of problem (1)-(4) are integers and satisfy the following conditions: $0<\underline{c}_j\le\overline{c}_j, 0\le\underline{a}_{ij}\le\overline{a}_{ij}, 0<\underline{b}_i\le\overline{b}_i,\ d_j>0, \left(i=\overline{1,m};\ j=\overline{1,N}\right)$ are the prescribed integers. In addition, the following natural conditions must be satisfied for the coefficients of problem (1)-(4):

$$\sum_{j=1}^{N}\underline{a}_{ij}d_j>\overline{b}_i,(i=\overline{1,m}).$$

If this condition is not satisfied for all $i,(i=\overline{1,m})$, then the solution $X=(d_1,d_2,\dots,d_N)$ will be the best, i.e. optimal solution to problem (1)-(4). It should be noted that if for some i_* the following inequality holds

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www.icp.az/2019/1-07.pdf

$$\sum_{j=1}^{N} \overline{a}_{i_* j} d_j \leq \overline{b}_{i_*},$$

this is not a limitation for system (2) and can be omitted. Therefore, here we assume that for problem (1)-(4) the above conditions are met.

Note that problem (1)-(4) can be referred to as a mixed-integer programming problem with interval data, or an interval mixed-integer programming problem.

It is pertinent to note that problem (1)-(4) is a generalization of: 1) Boolean programming problems; 2) integer programming problems; 3) interval Boolean programming problem; 4) interval mixed-integer programming problem; 5) interval mixed-Boolean programming problem; 6) linear programming problems; 7) interval linear programming problem. Because: 1) in cases where the ends of the intervals coincide, $n = N$ and $d_j = 1,\ \left(j = \overline{1, N}\right)$, we get a Boolean programming problem; 2) if $n = N$ and the ends of the intervals coincide, we get an integer programming problem; 3) if $d_j = 1,\ \left(j = \overline{1, N}\right)$ and $n = N$ we get an interval Boolean programming problem; 4) if $n = N$ we get an interval mixed-integer programming problem; 5) if $d_j = 1,\ \left(j = \overline{1, N}\right)$ and $n < N$ we get an interval mixed-Boolean programming problem; 6) if the ends of the intervals coincide and $n = 0$ we get a linear programming problem; 7) if $n = 0$ we get an interval linear programming problem.

Note that problem (1)-(4) is in the NP-complete class, because all the above special cases of this problem, except linear programming problems, are NP-complete, in other words, difficult to solve [1, p.123; 2, p.78]. Various classes of interval integer programming problems were investigated and specific methods were developed in [3–9, etc.]. And in [10, 11, 12, etc.], interval linear (non-integer) programming problems were investigated.

It should be noted that, as far as we know, the problem of mixed-integer programming with interval data has not yet been investigated. This may be due to the complexity of developing methods for their exact or approximate solution.

In this paper, new notions of admissible, optimistic, pessimistic, suboptimistic and subpessimistic solutions for problem(1)-(4) are introduced and an algorithm for its approximate solution is developed. In this case, the authors used the principles of interval computing introduced in [13].

2. Problem statement

First, we introduce some applied values of considered model (1)-(4). Suppose that a company produces N kinds of goods. Let of them n types $(n \leq N)$ be piece goods, and for $N - n$ types - non-piece

Suppose that for the production of these goods m types of resources are allocated, which are in the interval $\left[\underline{b}_i, \overline{b}_i\right]$, $(i = \overline{1, m})$. In addition, suppose that for the production of a unit of the j-th product, an expenditure is required, which is included in the interval $\left[\underline{a}_{ij}, \overline{a}_{ij}\right]$ $\left(i = \overline{1, m};\ j = \overline{1, N}\right)$. Let us assume that the profit from the sale of each j-th unit of goods is in the interval $\left[\underline{c}_j, \overline{c}_j\right], \left(j = \overline{1, N}\right)$. Let it be required to find such quantities of the planned types of piece and non-piece goods, for which the total expenses did not exceed the allocated resources $\left[\underline{b}_i, \overline{b}_i\right]$, $(i = \overline{1, m})$, respectively. In this case, the total profit was maximum. If we take the unknowns x_j $\left(j = \overline{1, N}\right)$, where $0 \leq x_j \leq d_j$, $\left(j = \overline{1, N}\right)$ and x_j are integers, $\left(j = \overline{1, n}; n \leq N\right)$, we get model (1)-(4).

It should be noted that in the areas of production where goods are produced, some of which must be single-piece, considered model (1)-(4) takes place.

3. Theoretical substantiation of the method

Before presenting the method for solving problem (1)-(4), we introduce some definitions. These definitions are an extension of the definitions introduced in [14] by the authors of this paper for mixed-integer programming problems.

Definition 1. An admissible solution to problem (1)-(4) is some vector$X = (x_1, \dots, x_N)$ which satisfies system (2)-(4) for $\forall\, a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ and $\forall b_i \in [\underline{b}_i, \overline{b}_i]$, $(i = \overline{1,m}; j = \overline{1,N})$.

This shows that in order to obtain the optimal solution and the maximum value of the objective function, it is necessary to ensure the condition of not exceeding the sum of some different intervals from a given interval, at the same time achieving the maximum amount of the corresponding intervals. It is obvious that it is almost impossible to satisfy these conditions. Therefore, we introduce the following definitions.

Definition 2. Let us call the admissible solution $X^{op} = (x_1^{op}, x_2^{op}, \dots, x_N^{op})$ the optimistic solution to problem (1)-(4), if for $\forall b_i \in [\underline{b}_i, \overline{b}_i]$, $(i = \overline{1,m})$ the relations $\sum_{j=1}^{N} \underline{a}_{ij} x_j^{op} \le b_i$, $(j = \overline{1,N})$ are satisfied and the value of the function

$$f^{op} = \sum_{j=1}^{N} \overline{c}_j x_j^{op}$$

will be maximum.

Definition 3. Let us call the admissible solution $X^p = (x_1^p, x_2^p, \dots, x_N^p)$ the pessimistic solution to problem (1)-(4), if for $\forall b_i \in [\underline{b}_i, \overline{b}_i]$, $(i = \overline{1,m})$ the relationships $\sum_{j=1}^{N} \overline{a}_{ij} x_j^p \le b_i$, $(j = \overline{1,N})$ are satisfied and the value of the function

$$f^p = \sum_{j=1}^{N} \underline{c}_j x_j^p$$

will be maximum.

From the last two definitions, it follows that the problem of finding optimistic and pessimistic solutions to problem (1)-(4) is also in the NP-complete class, since special cases of this problem are NP-complete. For this reason, we have developed **methods for the approximate solution** of the interval mixed-integer programming problem and called them suboptimistic and subpessimistic solutions.

When developing a method for constructing suboptimistic and subpessimistic solutions, we will use the economic interpretation of problem (1)-(4) introduced in the problem statement section.

Suppose some j-th item is being produced. Then the required resource from the allocated shared resource $[\underline{b}_i, \overline{b}_i], (i = \overline{1,m})$ will be in the interval $[\underline{a}_{ij}, \overline{a}_{ij}], (i = \overline{1,m}; j = \overline{1,N})$. At the same time, the profit from this j-th product must be in the interval $[\underline{c}_j, \overline{c}_j], (j = \overline{1,N})$. Obviously, the profit of the objective function for each unit of expenditure for the -th goods $(j = \overline{1,N})$ will be at least

$$\min_i \frac{[\underline{c}_j, \overline{c}_j]}{[\underline{a}_{ij}, \overline{a}_{ij}]} = \frac{[\underline{c}_j, \overline{c}_j]}{\max_i [\underline{a}_{ij}, \overline{a}_{ij}]}, (j = \overline{1,N}) \tag{5}$$

From this relationship, it follows that it is necessary to produce such a product with the number j_*, for which (5) will be the maximum:

$$\max_j \frac{[\underline{c}_j, \overline{c}_j]}{\max_i [\underline{a}_{ij}, \overline{a}_{ij}]} = \frac{[\underline{c}_{j_*}, \overline{c}_{j_*}]}{\max_i [\underline{a}_{ij_*}, \overline{a}_{ij_*}]} \tag{6}$$

Relationship (6) shows that in order to build suboptimistic and subpessimistic solutions, the number j_* should be determined, respectively, from the following relationships:

$$\max_j \frac{\overline{c}_j}{\max_i \underline{a}_{ij}} = \frac{\overline{c}_{j_*}}{\max_i \underline{a}_{ij_*}} \tag{7}$$

$$\max_j \frac{\underline{c}_j}{\max_i \overline{a}_{ij}} = \frac{\underline{c}_{j_*}}{\max_i \overline{a}_{ij_*}} \tag{8}$$

Obviously, when building suboptimistic and subpessimistic decisions based on criteria (7) and (8), it is strictly necessary to take into account the fact that the interval includes the number j_*, i.e. $j_* \in I = [1, \dots, n]$ or $j_* \in R = [n+1, n+2, \dots, N]$.

Considering these circumstances, in this paper we have developed two methods for building suboptimistic and subpessimistic solutions to problem. (1)-(4).

In the first method, if $j_* \in R$ and assigning d_{j_*} to the unknown x_{j_*} is impossible, then x_{j_*} we take the best possible value, while the remaining unknowns obviously take the value zero.

In the second method, if $j_* \in R$ and assigning d_{j_*} to the unknown x_{j_*} is impossible, then we fix all found values up to this point, and for the remaining numbers $j \in I$ we take $x_j \coloneqq 0$ and build a linear programming problem of a smaller dimension for x_j, where $j \in R$. Having solved the resulting linear programming problem of a smaller dimension, we add the obtained solutions to the previously fixed ones.

Note that in both methods at the beginning of the solution process we take $X^{so} = (0,0,\dots,0)$ and $X^{sp} = (0,0,\dots,0)$, where X^{so} and X^{sp} omean building suboptimistic and subpessimistic (i.e. approximate) solutions.

Principles of the first method for building a suboptimistic solution.

Assume $S := \emptyset$. Here S is many numbers of found and fixed unknowns. Obviously, two cases should be considered.:

Case 1. If $j_* \in I$, then x_{j_*} can take integer values from the interval$[0, d_{j_*}]$. Therefore, we take $x_{j_*} \coloneqq \min\left\{d_{j_*}; \min_i \left[\frac{b_i}{\underline{a}_{ij_*}}\right]\right\}$. Here $b_i \in [\underline{b}_i, \overline{b}_i]$, $(i = \overline{1,m})$ are fixed and [z] mean the integer part of the number z. First we take $b_i = \overline{b}_i$, $(i = \overline{1,m})$. Then we take $b_i := b_i - -\underline{a}_{ij_*} \cdot x_{j_*}, (i = \overline{1,m})$, $I \coloneqq I \backslash \{j_*\}$, $S \coloneqq S \cup \{j_*\}$ and the process of building continues.

Case 2. If $j_* \in R$, then x_{j_*} takes the value $x_{j_*} \coloneqq \min\left\{d_{j_*}; \min_i \left(\frac{b_i}{\underline{a}_{ij_*}}\right)\right\}$.

In this case, we should take into account 2 options:

1) If $d_{j_*} \le \min_i \left(\frac{b_i}{\underline{a}_{ij_*}}\right)$, then we take $x_{j_*} := d_{j_*}$, $b_i := b_i - \underline{a}_{ij_*} \cdot x_{j_*}, (i = \overline{1,m})$, $R \coloneqq R \backslash \{j_*\}$, $S \coloneqq S \cup \{j_*\}$ and the process continues.

2) If $d_{j_*} > \min_i \left(\frac{b_i}{\underline{a}_{ij_*}}\right)$, then $x_{j_*} \coloneqq \min_i \left(\frac{b_i}{\underline{a}_{ij_*}}\right)$, $(i = \overline{1,m})$, $b_i := b_i - \underline{a}_{ij_*} \cdot x_{j_*}, (i = \overline{1,m})$, $R \coloneqq R \backslash \{j_*\}$, $S \coloneqq S \cup \{j_*\}$ and the solution process is completed, since one of the right-hand sides will be equal to zero.

Continuing the process of building a suboptimistic solution$X^{so} = (x_1^{so}, x_2^{so}, \dots, x_N^{so})$ omeans finding the next number j_* from criterion (7). And if $I = \emptyset$ and $R = \emptyset$, then the process of building a suboptimistic solution is completed. It should be noted that building a sub-pessimistic solution $X^{sp} = (x_1^{sp}, x_2^{sp}, \dots, x_N^{sp})$ is carried out in a similar manner, using criterion (8).

Principles of the second method for building a suboptimistic solution.

In this method, Case 1 of the above first method remains unchanged. And the second case also remains valid only with the following alteration:

Assume $j_* \in R$, where j_* has been found from criterion (7). Besides, x_{j_*} has to take a smaller value than d_{j_*}. Then for j, $(j \in I)$ we take $x_j \coloneqq 0$. Next, we build a linear programming problem for the other variables of x_j, where $j \in R$.

Having solved the resulting linear programming problem of a smaller dimension, we obtain the optimal solution. Finally, we add the obtained coordinates of the solutions to the previously fixed coordinates.

Building a sub-pessimistic solution by the second method is carried out in a similar manner, using criterion (8).

The results of the computational experiments by both methods are presented below.:

In order to estimate the errors of suboptimistic and subpessimistic values, which have been built by the above methods, of objective function (1) from optimistic and pessimistic values, we use the following formulas:

$$\delta_{so}^1 \le \frac{\overline{f}_{op}-f_{so}^1}{\overline{f}_{op}},\ \delta_{so}^2 \le \frac{\overline{f}_{op}-f_{so}^2}{\overline{f}_{op}},\quad \delta_{sp}^1 \le \frac{\overline{f}_{p}-f_{sp}^1}{\overline{f}_{p}},\ \delta_{sp}^2 \le \frac{\overline{f}_{p}-f_{sp}^2}{\overline{f}_{p}}.$$

Here δ_{so}^1, δ_{so}^2, δ_{sp}^1, δ_{sp}^2 are the relative errors of suboptimistic and subpessimistic values from optimistic and pessimistic ones, respectively, f_{so}^1, f_{so}^2, f_{sp}^1, f_{sp}^2 are the csuboptimistic and subpessimistic values of the objective function obtained by methods 1 and 2, respectively. Notation $\overline{f}_{op}, \overline{f}_p$ means the optimal values of the objective function of the corresponding linear programming problems, i.e. the case of $n = 0$.

4. Results of the computational experiments

In order to determine the effectiveness of the developed methods, numerous computational experiments have been conducted for problems of various dimensions. The coefficients of these problems are random two- or three-digit numbers, satisfying the following conditions:

I. $0 \le \underline{a}_{ij} \le 99,\ 1 \le \overline{a}_{ij} \le 99,\ 1 \le \underline{c}_j \le 99,\ 1 \le \overline{c}_j \le 99,\ (j = \overline{1,N})$.

II. $0 \le \underline{a}_{ij} \le 999,\ 1 \le \overline{a}_{ij} \le 999,\ 1 \le \underline{c}_j \le 999,\ 1 \le \overline{c}_j \le 999,\ (j = \overline{1,N})$.

$\underline{b}_i := [\frac{1}{3}\sum_{j=1}^{N} \underline{a}_{ij} \cdot d_j],\ \overline{b}_i := [\frac{1}{3}\sum_{j=1}^{N} \overline{a}_{ij} \cdot d_j]$.

In all experiments, it is assumed that $d_j = 10,\ (j = \overline{1,N})$. The results are presented in the following tables. Note that for each dimension, 5 different problems are solved.

Table 1

Results of the solved problems with two-digit coefficients ($N = 500; n = 300; m = 10$)

№	1	2	3	4	5
$\overline{f}_{op}$	229490.527	227378.366	223084.763	224916.745	219829.074
f_{so}^1	218541.098	216886.032	213574.351	209878.203	209755.172
f_{so}^2	219348.833	219894.311	215927.731	214570.261	209821.538
δ_{so}^1	0.048	0.046	0.043	0.067	0.046
δ_{so}^2	0.044	0.033	0.032	0.046	0.046
k_{so}	121	112	99	111	105
$\overline{f}_p$	140399.242	141841.900	139481.812	137560.440	135850.227
f_{sp}^1	136751.500	136840.219	135128.138	133784.408	130227.312
f_{sp}^2	136751.500	137212.800	135986.897	133789.933	130356.455
δ_{sp}^1	0.026	0.035	0.031	0.027	0.041
δ_{sp}^2	0.026	0.033	0.025	0.027	0.040
k_{sp}	139	144	131	139	139

Table 2

Results of the solved problems with two-digit coefficients $N = 1000$; $n = 600$; $m = 10$)

№	1	2	3	4	5
$\overline{f}_{op}$	459128.212	452971.002	444381.328	450934.920	444367.018
f_{so}^1	439904.898	424629.000	424689.756	438604.000	426053.607
f_{so}^2	441019.333	427114.000	426761.500	439006.000	426728.970
δ_{so}^1	0.042	0.063	0.044	0.027	0.041
δ_{so}^2	0.039	0.057	0.040	0.026	0.040
k_{so}	208	229	211	222	233
$\overline{f}_{p}$	278280.833	281824.033	280697.887	278230.307	274328.993
f_{sp}^1	274655.320	276548.323	272920.850	273018.806	269465.714
f_{sp}^2	274974.000	276752.000	273628.600	273109.188	269480.318
δ_{sp}^1	0.013	0.019	0.028	0.019	0.018
δ_{sp}^2	0.012	0.018	0.025	0.018	0.018
k_{sp}	268	273	267	281	278

Table 3

Results of the solved problems with three-digit coefficients ($N = 500$; $n = 300$; $m = 10$)

№	1	2	3	4	5
$\overline{f}_{op}$	2078143.461	2048008.113	2011136.079	2036905.014	1996025.589
f_{so}^1	1972900.690	1939092.875	1895650.579	1923955.994	1861808.700
f_{so}^2	1988004.667	1969829.194	1907118.588	1932020.000	1870346.336
δ_{so}^1	0.051	0.053	0.057	0.055	0.067
δ_{so}^2	0.043	0.038	0.052	0.051	0.063
k_{so}	124	122	110	116	112
$\overline{f}_{p}$	1415717.044	1428348.507	1398445.814	1383115.720	1364663.837
f_{sp}^1	1373239.196	1381478.505	1375497.846	1346816.048	1322974.225
f_{sp}^2	1373665.135	1390936.884	1383093.878	1353101.464	1339018.991
δ_{sp}^1	0.030	0.033	0.016	0.026	0.031
δ_{sp}^2	0.030	0.026	0.011	0.022	0.019
k_{sp}	137	144	132	138	141

Table 4

Results of the solved problems with three-digit coefficients ($N = 1000$; $n = 600$; $m = 10$)

№	1	2	3	4	5
$\overline{f}_{op}$	4167732.651	4072629.011	4005598.048	4103210.997	4027310.198
f_{so}^1	3931821.231	3774727.636	3777433.362	3938478.406	3786266.718
f_{so}^2	3935901.500	3858809.333	3777544.462	3953751.000	3823791.095
δ_{so}^1	0.057	0.073	0.057	0.040	0.060
δ_{so}^2	0.056	0.053	0.057	0.036	0.051
k_{so}	225	249	224	235	244
$\overline{f}_p$	2807549.865	2842500.397	2828228.952	2805374.485	2770284.307
f_{sp}^1	2765745.846	2783400.329	2739368.955	2764326.022	2686987.913
f_{sp}^2	2768456.722	2787905.543	2745060.867	2768239.877	2701497.674
δ_{sp}^1	0.015	0.021	0.031	0.015	0.030
δ_{sp}^2	0.014	0.019	0.029	0.013	0.025
k_{sp}	266	277	267	282	282

5. Conclusions

Based on these tables, the following conclusions can be made: the suboptimistic values of the objective function in problems (1)-(4), as well as the sub-pessimistic ones obtained by methods 1 and 2 in this paper, are not strictly different from each other. Since in the first method does not use the linear programming apparatus, the first method can be considered more practical than the second. And in the second method it is necessary to solve part of the problem as a linear programming problem. Therefore, if time allows, then the use of the second method can be considered higher quality. It should be noted that the mean relative errors of suboptimistic and subpessimistic values from optimistic and pessimistic do not exceed 2-7% and 1-3%, respectively. In addition, the number of remaining variables for the linear programming problem is 20% on average. These conclusions once again confirm the high quality and applicability of the developed method for real applied problems.

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