Informatics and Control Problems 39 Issue 1 (2019) 64-70
journal homepage: www.icp.az

Mathematical modeling of the performance of aerobatic figures by a group of drones during joint flight

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ARTICLE INFO

Article history:
Received 06.03.2019
Received in revised form 15.04.2019
Accepted 22.05.2019
Available online 12.06.2019

Keywords:
Mathematical model
Drone Unmanned Aerial Vehicle
Aerobatic figure
System of systems
Multi-agent system

ABSTRACT

A mathematical model of the joint movement of a group of drones is developed. To this end, a methodological approach is developed to simulate tasks for a joint flight of a group of drones in accordance with the System of Systems ideology. Other drones, members of a group flight, are considered dynamic obstacles. In accordance with the developed methodology, the structure of the information exchanged between the drones during a joint flight is proposed. In addition to the initial model of the joint movement of drones, the particular cases of joint flight of drones when performing four aerobatic figures: "Arrangement in a line", "Circle", "Movement of drones along a circle" and "Complex aerobatic figure" was investigated and described.

1. Introduction

Studies show that in the operation of unmanned aerial vehicles (drones), group flight is increasingly being used - grouping drones in "flocks" to achieve a common goal. The flock model is currently the most promising technology for group use of special-purpose drones [1,2].

To describe such a "flock", scientific literature often uses such concepts as "Multi-agent system" (MAS) [3] and "System of systems" (SoS) [4]. In both cases, it is a form of organization of several interacting independent intellectual subsystems (agents) for the purpose of performing a specific task that is difficult or impossible to solve by means of a single subsystem (agent). The principal difference between the two is that the components of MAS are programs, and the components of SoS are objects of different nature (technical, technological, biological, social, etc.). At the same time, if a SoS is formed by software objects, then it is a MAS. In other words, MAS are a particular type of SoS, i.e. a software SoS [5].

The idea of using group flight of drones in the military field is not new either, it provides for joint activities of interacting drones in order to increase the likelihood of successful execution of tasks. In this case, each drone acts as an independent aircraft in the execution of a task, making a decision based on the information exchanged within the group, which in the general case may be incomplete.

Literature covers various aspects of drone control. Many studies are devoted to the problems of controlling one selected drone, e.g. [3-8], although there are also studies devoted to the issues of joint flight of a group of drones [1, 9].

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www.icp.az/2019/1-08.pdf

2. Problem statement

In this paper, we assume that each drone within a group that performs a joint flight will regularly exchange information with other drones, being controlled independently. It is also assumed that in all sessions of information exchange for decision-making, each of the drones receives all the necessary data about other drones participating in the joint flight.

Solving the problem of automating the control of a group flight should ultimately lead to the implementation of certain tasks, including:

- cargo delivery;
- photo/video shooting in the visible or infrared range;
- relaying of electromagnetic signals;
- performing aerobatic figures.

It is obvious that the implementation of these tasks requires the involvement of additional technical equipment. Having received one or another task or several tasks at the same time, a group of drones flies to the place where the tasks are performed, carrying them out according to a given schedule. The regulations may provide for the order of execution of tasks, e.g. "sequentially", "according to a given schedule," etc. For simplicity, we will consider the sequential execution of tasks.

It should be noted that in all cases an integral part of the problem of control when performing tasks is controlling the flight of the drone. When performing a task, drones must fly to the scene, complete the task and return to the designated landing site. Consequently, the flight trajectory of drones can be divided into the following conventional parts:

(1) flight to the place of mission;
(2) reconfiguration of the flight in order to accomplish the task (getting into position, differentiation of speed and/or direction of movement, etc.);
(3) executing the task;
(4) returning to the starting position, or executing the next task (then the process starts from point “1”).

It is assumed that the trajectory of each drone participating in a group flight is determined on the basis of the trajectory of the entire group, the individual parameters of the drone, the schedule and content of the tasks, terrain and movement dynamics. Thus, the problem of developing a decision-making system comes down to the development of a methodology for solving tasks "1"-"4".

3. Basic concepts and notation

Usually, an organized group of drones begins executing tasks within a certain geographical area. Although the generally accepted system for specifying the coordinates of objects on the terrain is geographical latitude and longitude, the use of transformation formulas [10] will make it possible to express them with a certain degree of accuracy by Cartesian coordinates specified in some rectangular coordinate system. For simplicity of presentation of the formulas, we will assume in the following paragraphs that the real geographical coordinates of the current location of the drones are given in some rectangular coordinate system.

Drones have a number of performance characteristics that must be considered when calculating the trajectory of their movement. Such restrictions include, for instance, the maximum permissible speed, the maximum permissible acceleration, etc. As mentioned above, in this paper, we consider the case of sequential tasks. For definiteness, we will enumerate the tasks:

1. Flyby.
2. Group performance of aerobatic figures.
3. Delivery of goods, etc.

The movement of drones constitutes the main part of every task, therefore, the main control problem is to determine the trajectory of each drone and control its movement along this trajectory. The state of the drone is described by generalized coordinates. By generalized coordinates we understand a set of data of nine indicators, which is a set of ordinary rectangular coordinates, components of the velocity vector and components of the acceleration vector. Each such task has its own generalized input and output coordinates - the condition for the start and end of the task.

When calculating the trajectory of drones, it is necessary to take into account the presence of stationary obstacles that may depend on the terrain, individual buildings, etc. However, in this study, stationary obstacles will not be taken into account, assuming that there are no such obstacles within the geographical area under investigation.

Let the group have $N+1$ drones. We assume that the drones have base numbers from 0 to N. When they receive a group of tasks, they are assigned work numbers. The numbering rule (assignment of work numbers) is given Section 4.

Each task implies some initial arrangement of the group of drones, the end point, which we call start, and the final boundaries of the task. In this case, each final boundary is the initial boundary of the next task. This concept was introduced so that, if necessary, the drone had the opportunity to smoothly change its coordinates and proceed to the next task on the list. Thus, the concept of "boundary" does not always coincide with the start or end point of the trajectory. The mechanism for determining the boundaries for each task will be described further.

Regardless of the sequence of tasks that will be received by the group of drones, it is assumed that at the initial moment they are located in the start points with known coordinates. We denote these coordinates by $\{(x_{i00}, y_{i00}, z_{i00}), i = 0,1, \dots, N\}$. Here, the index i is the base number of the drone and it is of a formal nature.

4. Dynamic drone model

In this paper, we consider propeller drones of tricopter, quadcopter, hexacopter and octocopter types. These drones are controlled by their propeller engines and have no other controls. Thus, the movement and spatial orientation is controlled only by forces and torques generated by propellers. In principle, drone's direction of movement does not depend on its spatial orientation. Therefore, when modeling the dynamics of a drone, we will neglect its spatial orientation and consider them as material points, although in this section we will consider the problem of modeling the dynamics of an individual drone. To preserve the identity of the notation, we will emphasize everywhere that the coefficients included in mathematical formulas can be different for different drones.

For definiteness, we denote the initial moment of drone's motion by t_0 and will simulate the drone's dynamics for $t \geq t_0$. Let the coordinates of the i-th drone be $x_i(t), y_i(t), z_i(t)$, where $i = 0,1, \dots, N$. Let m_i denote the mass of the i-th drone. Then the system of equations of their movement will look as follows:

$$\begin{cases} m_i x_i''(t) = -\alpha_{xi}(\varphi) x_i'(t) + u_{xi}(t) \\ m_i y_i''(t) = -\alpha_{yi}(\varphi) y_i'(t) + u_{yi}(t) \\ m_i z_i''(t) = -\alpha_{zi}(\varphi) z_i'(t) + u_{zi}(t) \end{cases} \tag{1}$$

where $\alpha_{xi}(\varphi)$, $\alpha_{yi}(\varphi)$, $\alpha_{zi}(\varphi)$ are air drag coefficients in the directions of axis Ox, Oy and Oz, respectively, which depend on the orientation angle φ of the drone relative to the direction of flight;

(u_{xi}, u_{yi}, u_{zi}) is the control vector of the force generated by the power plants of the i-th drone at the moment of time t.

Let at the moment t_0, the coordinates and velocity components of all drones are given:

$$x_i(t_0) = x_{i00}, \quad y_i(t_0) = y_{i00}, \quad z_i(t_0) = z_{i00}, \tag{2}$$
$$x_i'(t_0) = x_{i10}, \quad y_i'(t_0) = y_{i10}, \quad z_i'(t_0) = z_{i10}. \tag{3}$$

If at the moment t_0 the drones are at the initial positions on the ground, then obviously, $x_{i10} = y_{i10} = z_{i10} = 0$.

Depending on the tasks, controlling the movement of a drone implies its arrival at a certain time at a certain place. In other words, control must ensure that the boundary condition at the end point of the trajectory is fulfilled. Assume that the i-th drone should appear in some point with coordinates $(x_{i0m}, y_{i0m}, z_{i0m})$ at some final moment $t = t_m$. Then the boundary conditions at $t = t_m$ can be specified in the form:

$$x_i(t_m) = x_{i0m}, \quad y_i(t_m) = y_{i0m}, \quad z_i(t_m) = z_{i0m}. \tag{4}$$

It is obvious that (1)-(4) are a control task that can be executed in different ways depending on the conditions caused by the technical capabilities of drones, terrain, task characteristics, etc.

5. Mathematical model of the "flyby" task

This task involves the movement of a group of drones along a certain trajectory along its given set of nodal points. The main requirement for the calculated trajectory is the minimality of its length. As mentioned above, the calculation of time will be conducted from some point t_0. Obviously, it is necessary to distinguish the concept of the trajectory of a group of drones and the trajectory of individual drones belonging to this group. When a task is set for a group, naturally, the trajectory of the group must be set, and further, based on it, the trajectories of each drone should be determined separately.

Let the nodal points of the group trajectory be given by the following sequence of spatial points: $M = \{M_m(x_m, y_m, z_m), m = 1,2, \dots, m_0\}$, where $M_1(x_1, y_1, z_1)$ is the first nodal point along the trajectory after the start of movement, m_0 is the number of the last nodal point.

The center of the group of drones is the spatial point (x_0, y_0, z_0) with coordinates $\left(\frac{1}{N+1}\sum_{i=0}^{N} x_{i00}, \frac{1}{N+1}\sum_{i=0}^{N} y_{i00}, \frac{1}{N+1}\sum_{i=0}^{N} z_{i00}\right)$.

The trajectory of the group is the broken line with segments $\{L_m, m = 1,2, \dots, m_0\}$ connected by the nodal points $M_{m-1}(x_{m-1}, y_{m-1}, z_{m-1})$ and $M_m(x_m, y_m, z_m)$. Let at the moment t_0 the coordinates and velocity components of all drones are given by formulas (2)-(3). Based on the data of the first segment L_1, drones are assigned numbers that they retain until the number of drones in the group changes. The numbering is as follows:

(1) A drone is selected, the coordinates of which are closest to the line containing the segment L_1. If there are several of such drones, then the one that is closest to the point (x_1, y_1, z_1) is selected. If there are several of such drones as well, then the one with the minimum base number is selected from among them. This drone is assigned the number 0.

(2) Then, the points are ordered in order of distance from the plane $P(x, y, z) = 0$, where

$$P(x, y, z) \equiv (y_{m-1}(z_m - 1) - y_m(z_{m-1} - 1))x + (x_m(z_{m-1} - 1) - x_{m-1}(z_m - 1))y + (x_{m-1}y_m - x_m y_{m-1})(z - 1).$$

Drones are numbered depending on the positivity or negativity of the number $P(x_{i00}, y_{i00}, z_{i00})$, through $+1, +2, \dots, i_{(+)}$ or $-1, -2, \dots, -i_{(-)}$, where $i_{(+)}$ and $i_{(-)}$ are the number of drones with positive and negative numbers, respectively.

The coordinates of the arrival of the i-th drone in the vicinity of the point (x_m, y_m, z_m), $m = 1,2, \dots, m_0$ are calculated from the formulas:

$$\begin{cases} x_{i0m} = i(y_m - y_{m-1}), \\ y_{i0m} = -i(x_m - x_{m-1}), \\ z_{i0m} = 0, \end{cases} \quad (5)$$
$$i = \pm 1, \pm 2, \dots, \pm i_{(\pm)}.$$

It should be noted that the boundary conditions for determining the trajectory of drones for each subsequent interval (M_{m-1}, M_m) are written in the same way. Here, conditions (3) for the next trajectory interval are determined as the values of the velocity components of the corresponding drones at the end of the previous trajectory interval.

Conditions for limiting the maximum allowable speed and acceleration of drones during flight may be as follows:

$$\left(x_i'(t)\right)^2 + \left(y_i'(t)\right)^2 + \left(z_i'(t)\right)^2 \le v_i^2, \quad (6)$$
$$\left(x_i''(t)\right)^2 + \left(y_i''(t)\right)^2 + \left(z_i''(t)\right)^2 \le a_i^2, \quad (7)$$

where v_i and a_i are the maximum allowable speed and the maximum allowable acceleration of of the i-th drone, respectively.

In this study, dynamic obstacles are other drones - members of a group flight. If we assume that each next drone should in the calculation avoid colliding with drones with numbers smaller in modulus, then the conditions for avoiding dynamic obstacles can be written in the following form:

$$(x_i(t) - x_k(t))^2 + (y_i(t) - y_k(t))^2 + (z_i(t) - z_k(t))^2 \ge \delta_0^2, k < i, \quad (8)$$

here δ_0 is the minimum allowable distance between drones in a group flight.

Obviously, there are infinitely many trajectories that satisfy conditions (1)-(8). To select one reasonable (optimal) trajectory among all possible, let us set the condition of minimality of the trajectory length:

$$\int_{t_{m-1}}^{t_m} \sqrt{1 + \left(x_i'(t)\right)^2 + \left(y_i'(t)\right)^2 + \left(z_i'(t)\right)^2}\, dt \to min. \quad (9)$$

Thus, we have built a mathematical model of the problem of determining the trajectory of the i-th drone in a group flight.

6. Mathematical model of the "group execution of aerobatic figures" task

There is no clear definition of aerobatic figures for propeller drones. In this paper, by the aerobatic figures for a group of drones we mean:

- the arrangement of the drones in space at predetermined positions (e.g. along a line, a circle, etc.);
- movement of drones along predetermined trajectories (e.g. along a circle or along a zigzag trajectory, etc.);
- drones switching from one position to another.

Each aerobatic figure can be characterized by relevant parameters. It is clear that some of these parameters will be obvious in accordance with the nature of the figure. For instance, the arrangement of drones in space in predetermined positions implies zero velocity of drones in these positions. Some parameters may vary within certain limits. For instance, the movement of a drone along a given trajectory implies specifying linear velocity, etc. Since the characteristic parameters of executing aerobatic figures can differ greatly from those parameters that the drones have as they fly to the place of their execution, it is necessary to ensure the possibility of a smooth change of these parameters. For this purpose, the concept of "event line" is introduced:

The event line is a place in space where the drone begins to change its parameters, which allows, in the future, to perform the next task on the list.

A flight at the event line is analyzed at the task formulation stage. Therefore, if a smooth transition from one task to another is impossible, the system will warn the operator (the person who will formulate tasks for the group flight) about the infeasibility of the task.

Suppose that an aerobatic figure must be performed at the point in space $M_p(x_p, y_p, z_p)$. If the execution of aerobatic figures is preceded by the "flyby" task and $M_o(x_o, y_o, z_o)$ is the last nodal point in the "flyby" task, the event line is the area in space located between points M_o and M_p.

Let us consider a number of basic "aerobatic figures", which will be discussed in this study. We assume that a group of drones can perform more complex "aerobatic figures" that can be represented as combinations of these basic figures.

6.1. "Arrangement in a line". This figure implies that drones, flying up to a certain line, will hover in the air. The main parameters of this figure are (Fig. 1.):

$M_p(x_p, y_p, z_p)$ – place of arrangement;
$N_p(x_{np}, y_{np}, z_{np})$ – direction vector of the line of arrangement;
$D_p(x_{dp}, y_{dp}, z_{dp})$ – distance between the drones in the line;
T_p – duration of execution of the figure;
$V_p = 0$ – formal speed of the movement of drones along the line of arrangement. This parameter is introduced to unify the main descriptive parameters of the executed figure.

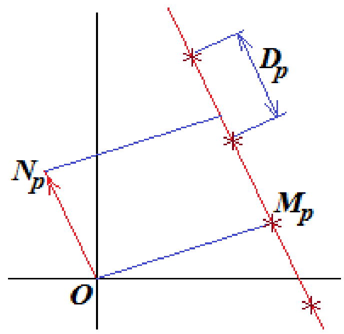

Fig. 1 Geometry of the "arrangement in a line" figure

6.2. "Circle". This figure implies that the drones, flying up, take the points evenly spaced along a certain circle. The main parameters of this figure are (Fig.2.):

$M_p(x_p, y_p, z_p)$ – center of the circle;
$N_p(x_{np}, y_{np}, z_{np})$ – direction vector of the circle plane;
$D_p(x_{dp}, y_{dp}, z_{dp})$ – distance between the drones along the circle;
T_p – duration of execution of the figure;
V_p – speed of the movement of drones along the circle. It makes sense when performing the "movement of drones along a circle" figure. When performing the figure, the "circle" is taken as $V_p = 0$.

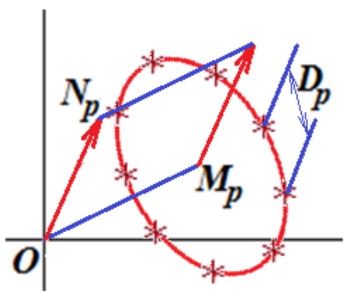

Fig. 2 Geometry of the "circle" figure

6.3. "Movement of drones along a circle". This figure implies that a group of drones first takes up positions along a certain circle. Then they all start going around the center of this circle, i.e. move along this circle. The main parameters of this figure are identical to the parameters of the "circle" figure. In this case, V_p, obviously, will be different from zero. The direction of movement along the trajectory is determined by the positivity or negativity of V_p relative to the vector N_p.

6.4. "Complex aerobatic figure". This type of aerobatic figures include version with changing

- place of arrangement – "parallel" transition of the group to a new location;

- direction vector of the line of arrangement or the circle plane – rotation of the group around a certain point;
- distance between the drones – transition of the drones in the group to a more rarefied or closer arrangement along a line or circle, if that is possible;
- speed of the movement of drones along the circle – increase or decrease of the speed of movement along the circle.

7. Conclusion

The main result of this study is the development of a mathematical model of the joint movement of a group of drones. To this end, a methodological approach is developed to simulate tasks for a joint flight of a group of drones based on the ideology of System of Systems. In accordance with the developed methodology, the structure of the information exchanged between the drones during a joint flight is proposed.

In addition to the initial model of the joint movement of drones, the particular cases of joint flight of drones when performing four aerobatic figures: “Arrangement in a line”, “Circle”, “Movement of drones along a circle” and “Complex aerobatic figure” was investigated and described.

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