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Modeling the penetration of cylindrical bodies into the ground by vibration

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ABSTRACT

The article provides a method of reporting the pile bearings by vibration and ensuring the pavement reliability in order to minimize environmental damage during the construction of agricultural hydraulic engineering installations. The frictional forces in the outer and inner surfaces of the cylinder caused by the interaction with the ground are taken into account. The rod is taken as elastic, and the soil as a viscoelastic medium. As a result, the expressions for contact surfaces of the media, the value of the necessary longitudinal load when releasing the stuck part of the column from vibration are obtained.

1. Introduction

Studies on the vibrations of tubular bodies in the deformable medium are of theoretical and practical interest. In particular, such problems include longitudinal oscillations of round rods recessed into the ground. The difficulty in solving this problem is due to the influence of the environment and the complexity of the contact conditions over the rod surface, especially when the frictional force is taken into account.

The first results, where the problems for the rods are formed and solved, were obtained in [1-4]. In particular, in [3], the environment is taken as Winkler and the equation of the longitudinal vibration of an elastic rod is derived for dry friction according to Coulomb's law, close to the classical one, i.e. to the second-order partial differential equation. In the present paper, by theoretical studies, the dynamic load required to release and clog the elongated elastic bodies of the circular section at a variable depth of the deepening into the ground is determined, taking into account the frictional forces and soil rheology.

2. Problem statement

Let us consider an elastic long hollow cylindrical rod of radius $r \in [R_1; R_2]$ in the homogeneous isotropic viscoelastic medium of thickness$\delta = R_2 - R_1$. Here R_1 is an internal and R_2 external radius of the rod.

The problem is set in three-dimensional linear formulations. The dependence of the stress σ_{ij} on the deformation ε_{ij} in the rod and the medium is taken in the form

$$\sigma_{ij}^{(m)} = L_m\left[\varepsilon^{(m)}\right] + 2M_m\left[\varepsilon_{ij}^{(m)}\right]; (m = 0; 1) \quad (1)$$

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$$\sigma_{ij}^{(m)} = M_m\left[\varepsilon_{ij}^{(m)}\right]; \quad (i \neq j;\ i';j = r,\theta,z).$$

where $m = 0$ corresponds to the rod and $m = 1$ to the medium: L_m and M_m are the linear operators. $L_0 = \lambda\,;\ M_0 = \mu\,;\ \ \lambda$ and μ are the elasticity constants of the material of the rod :

$$L_1(\zeta) = \lambda_1\left[\zeta(t) - \int_0^1 \Gamma_1(t-\xi)\zeta(\xi)d\xi\right]$$

$$M_1(\zeta) = \mu_1\left[\zeta(t) - \int_0^1 \Gamma_2(t-\xi)\zeta(\xi)d\xi\right]$$

λ_1 and μ_1 are elasticity constants of the medium: $\Gamma_1(t), \Gamma_2(t)$ are the kernels of the viscous elasticity operators.

Equations (1) of motion of the rod and the medium have the form

$$(L_m + 2M_m)(\Delta F_m) = \rho_m \partial^2 F_m/\ \partial t^2 \qquad (2)$$
$$M_m(\Delta Q_m) = \rho_m \partial^2 F_m/\ \partial t^2$$

where Δ is the Laplace operator in the cylindrical coordinates.

In the problems of releasing the stuck column, the material of the inner and outer parts of the pipe is assumed to be the same. With full clamping, the soil plug fills the entire internal cavity of the carbonized part of pipe, so the desired magnitudes do not depend on the angle, i.e. the axisymmetric problem in cylindrical coordinates is considered. The vector of replacements through the potentials F_m and $Q_ъ$ is defined by the formula

$$\vec{U}_m = gradF_m + rot[rot(Q_m\bar{e}_я]$$

Longitudinal oscillations of the rod is caused by the external forces f_1 and f_2 given on the surfaces of the $r = R_i, (i = 1,2)$. The initial conditions are assumed to be zero. The boundary conditions have the form:
under conditions of dry friction on the surfaces $r = R_i$

$$\sigma_{rr}^{(0)} = \sigma_{rr}^{(1)} + f_{ir}(z,t);\ \sigma_{rz}^{(0)} = -\eta\sigma_{rr}^{(0)} \qquad (3)$$
$$\sigma_{rz}^{(0)} = \eta\sigma_{rr}^{(1)} + f_{ir}(z,t);\ u_r^{(0)} = u_r^{(1)}$$

with rigid contact on surfaces $r = R_i$

$$\sigma_{rr}^{(0)} = \sigma_{rr}^{(1)} + f_{ir}(z,t);\ u_r^{(0)} = u_r^{(1)} \qquad (4)$$
$$\sigma_{rz}^{(0)} = \sigma_{rr}^{(1)} + f_{ir}(z,t);\ u_z^{(0)} = u_z^{(1)}$$

As is known, the coefficient η_0 of the friction in dynamics differs from static friction and its sign depends on the sign of the rate of relative slip of particles over the contact surface. Therefore, in the general case the problem under boundary conditions (3), (4) is a nonlinear boundary value problem.

3. Solution of the problem

We assume that the coefficient of friction η_0 in modulus coincides with the statistical coefficient of friction and keeps one or another sign during the time interval under study. In view of the imposed conditions, the coefficient η_0 can be considered constant and independent of the unknown functions, and the problem itself is reduced to the linear one. In the case of the ideal contact $\eta_0 = 0$ the external forces f_{ir} in general include a steady and dynamic normal pressure on the rod. f_{iz} is a tangential effects on the surface of the rod. The functions of external forces can be represented as

$$f_r(z,t) = \int_0^1 \begin{Bmatrix} \sin kz \\ -\cos kz \end{Bmatrix} dk \int_0^1 f_r^{(0)}(k;p)\exp(p\tau)\,d\tau.$$

$$f_z(z,t) = \int_0^1 \begin{Bmatrix} \cos kz \\ \sin kz \end{Bmatrix} dk \int_0^1 f_z^{(0)}\,(k;p)\exp(p\tau)\,d\tau. \tag{5}$$

Due to this, the potentials F_m and Q_m are also sought in the form

$$F_m = \int_0^d \begin{Bmatrix} \sin kz \\ -\cos kz \end{Bmatrix} dk \int_0^1 F_m^{(0)} \exp(p\tau)\,d\tau.$$

$$Q_m = \int_0^d \begin{Bmatrix} \cos kz \\ \sin kz \end{Bmatrix} dk \int_0^1 Q_m^{(0)} \exp(p\tau)\,d\tau. \tag{6}$$

Substituting (5), (6) into (2), for $F_m^{(0)}$ and $Q_ь^{(0)}$, we obtain the following ordinary differential equations

$$\frac{d^2F_m^{(0)}}{dr^2} + \frac{1}{r}\frac{dF_m^{(0)}}{dr} - \alpha_m^2 F_m^{(0)} = 0$$

$$\frac{d^2Q_m^{(0)}}{dr^2} + \frac{1}{r}\frac{dQ_m^{(0)}}{dr} - \beta_m^2 Q_m^{(0)} = 0$$

where

$$\alpha_m = \rho_m \rho^2 \,/\, L_m^{(0)} + k^2\,.$$
$$\beta_m = \rho_m \rho^2 \,/\, M_m^{(0)} + k^2\,.$$

$L_m^{(0)}$ and $M_m^{(0)}$ are the Laplace transforms of the viscos elasticity operators L_m and M_m.

The boundary conditions take the forms

$$\sigma_{rr.0}^{(0)} = \sigma_{rr.0}^{(1)} + f_{ri}(k,p).$$
$$\sigma_{rz.0}^{(0)} = \eta\sigma_{rr.0}^{(0)}\,.$$
$$\sigma_{rz.0}^{(0)} = \eta\sigma_{rr.0}^{(1)} + f_{rz}(kp), \quad u_{r.0}^{(0)} = u_{r.0}^{(1)}.$$
$$\text{at } i = 1\;\; r = R_i\,; \quad i = 2\;\; r = R_2\,.$$
$$\sigma_{rr.0}^{(1)} + f_{fi}^{(0)}(k,p), \quad u_{r.0}^{(0)} = u_{r.0}^{(1)}.$$
$$\sigma_{rz.0}^{(0)} = \sigma_{rz.0}^{(1)} + f_{rz}^{(0)}(r,p), \quad u_{z.0}^{(0)} = u_{z.0}^{(0)}$$

Solution of the equations bounded at $r = 0$ and $r = \infty$ is of the form

$$F_0^{(0)} = A_0 I_0(\alpha_0 r)\,; \qquad Q_0^{(0)} = B_0 I_0(\beta_0 r);$$
$$F_1^{(0)} = A_1 K_0(\alpha_1 r)\,; \qquad Q_1^{(0)} = B_1 K(\beta_1 r).$$

where $L_v(\zeta)$ and $K_v(\zeta)$ are the modified Bessel functions. The coefficients $A_1(p), B_1(p)$ are easily determined from the boundary conditions. The expressions of these coefficients are too cumbersomeness and are not given here. When studying wave processes in a rod, the arguments K_v of the modified Bessel functions are large (large values of p), therefore, expanding the functions $L_v(\zeta)$ and $K_v(\zeta)$ by power series and passing to the main parts, we find solutions to the problem in the soil and the rod. It is possible to separate the velocity of the Rayleigh surface wave

$$c_k = (\beta_1^2 + k^2)^2 - 4k^2\alpha_1\beta\,.$$

The speed of the longitudinal wave in the rod is determined by the expression that depends on the parameters of the rod, medium and on the coefficient of friction η_0

$$c = c_1\left[\sqrt{1+\alpha^2 c_1^2} + \alpha c_1\right]^1.$$

where

$$\alpha_1^2 = \rho_1\rho^2 \,/\, N_1^{(0)} + k^2 \;:\; \beta_1^2 = \rho\rho^2 / \, M_1^{(0)} + k^2 \;.$$

$$\alpha = \frac{R_1+R_2}{2}\frac{\eta_0}{2}\frac{\alpha_0^2 - 2\beta_0^2}{\beta_0^2}\frac{\mu_1\alpha_1}{\mu_0\beta_1^2}$$

$$\beta = \frac{R_1+R_2}{2}\frac{\eta_0}{2}\left(2+\frac{\mu_1}{\mu_0}\right)\frac{\alpha_0^2 - 2\beta_0^2}{\beta_0^2}$$

μ_0 and μ_1 are the coefficients of the transverse deformation of media: theoretically obtained expression c makes it possible to determine the frequencies of the vibration action on the columns, which ensures its release, when the longitudinal tensile force is used together. In this case, working in a resonance mode, the value of the longitudinal-tensile force can be reduced up to the weight of the column.

If the frequencies of vibration are incompatible with the frequency of wave propagation in the in the rod, the required value of the propagating force for releasing the column is determined from the expression [5].

$$Q = 2\pi R\int_0^{l(t)} \eta_0(f_1(z) + f_2(z))dz + P_k \,.$$

where P_k is a weight of the column:
$R = \frac{R_1+R_2}{2}$ – average radius of the rod:
$f_1(z)$ – forces of the soil resistance:
η_0 – friction coefficient:
$l(t)$ – depth of groove of the rod:

$$f_1(z) = \sum_{i=o}^{n} f_1(z) \;:$$

$$f_0(z) = N_1\frac{chkz}{shkl} - \frac{N_2 chk(z-l)}{shkl} - N_3 z + N_4 \;:$$

$$f_{i-1}(z) = \bar{c}_1 e^{kz} + \bar{c}_2 e^{-kz} + \int_0^1 F(\xi) shk(l-\xi)dz \;\;:$$

$$f_2(z) = \bar{c}_1 e^{\beta z} + \bar{c}_2 e^{-\beta z} + \frac{1}{\beta}\int_0^1 B_1(\xi) sh\beta(l-\xi)dt \;\;:$$

$\beta = \sqrt{\frac{b_7}{4b_0}}$; $N_i, \beta, \bar{c}_1$ are constants.

4. Conclusion

We propose the following formula (in the static case) to determine the values of the release force of a stuck column with sufficient accuracy in the engineering practice

$$Q = Aexp(\gamma_1 z)\big[\big(M(z) + T(z)\big)\cos\gamma s + M(z)\sin\gamma z\big].$$

where

$$M(z) = \frac{\bar{f}(z)}{2\gamma^2}\left(z - \frac{1}{\gamma}\right) : \quad T(z) = \frac{\bar{f}(z)}{2}\left(2z - \frac{1}{\gamma}\right):$$

$$D = \frac{Eh^2}{12(1-v^3)}: \quad \gamma = 3(1-v^2)/(Rh)^2 ;$$
$$\gamma_1 = \beta\gamma; \quad f(z) = f_1(z) - f(z);$$
$$h = R_2 - R_1 ; \quad R = 0.5(R_1 + R_2).$$

The studies show that static load is not transmitted to great depths, therefore the release from static stretching does not give the desired results. This circumstance further increases the importance of the application of dynamic vibro-exposure methods.

To free the stuck column, the Rayleigh wave propagating along the boundary of the two media plays an important role. As the calculations show, $c_p \approx 0.8c$. Thus, the frequency of vibro-exposure should be within $(0.8 \div 1)c$.

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