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On one method of block transfer of conditions for a system of three-step discrete processes connected only by boundary conditions

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ABSTRACT

The author considers a complex discrete process consisting of a large number of independent subprocesses that are interconnected only by boundary states. The state of the process is described by systems of linear three-step difference block structure equations. Schemes and appropriate formulas based on the idea of a block transfer of conditions that take into account the specifics of the Jacobian of the system and the weakness of the Jacobian matrix of the conditions of relationship between the subsystems are proposed. The results of numerical experiments obtained in solving the test problem are presented.

1. Introduction

The problem of calculating the state of discrete processes described by systems of linear three-step difference block structure equations [1] of high dimensionality with weak and arbitrary relationships between the values of the initial and final states of the subsystems is studied. Numerical schemes of solutions, corresponding formulas based on the developed algorithm for block transfer of conditions are proposed. They take into account the specifics of the Jacobian of the system and the weak filling of the Jacobian matrix of the conditions of relationships between the subsystems.

Such problems arise, for instance, when applying the difference approximation of mathematical models of multilink objects with lumped or distributed parameters of many continuous complex processes [2–6]. It should be noted that many discrete mathematical models encountered in practice are obtained using decomposition of complex processes by temporal or spatial variables into simpler subprocesses with previously known mathematical models or into subprocesses for which it is easy to construct them. The state of such processes in the general case is described by systems of linear differential equations with ordinary or partial derivatives, which are characterized by a high dimensionality, block structure, weakly and arbitrarily filled (sparse) coefficient matrices of non-separated boundary conditions.

The direct use of standard (known) methods for running boundary conditions to solve the difference boundary value problems under consideration is not effective [7–9]. As we know, taking into account the block structure of conditions for many other classes of computational problems can significantly accelerate their solution. For instance, in [10], in order to parallelize and accelerate the

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work with super-large matrices, they are reduced to a block-diagonal form. In [1, 11–13], a direct multistep method was proposed for solving second-order ODEs based on dividing the time interval into sub-intervals of the same size in which a set of solutions is generated simultaneously. In [12], this method was developed for the variable size of each block and for calculating the solution simultaneously at four points, but using a numerical interpolation approach. In contrast to these works, in the problem under consideration, the blocks are interconnected by non-separated conditions, and for solving the problem, a sweep method scheme is proposed that takes into account its block structure, and the solution method can be directly parallelized [14-16].

In contrast to the approaches proposed by the authors of [16, 17], in which the blocks have the same size and inversion of the matrices of each of the blocks is used, the solution method being iterative, in this paper the blocks of the system can have different sizes and matrix inversion is not used in the proposed formulas. The proposed method is not iterative, it allows simultaneous block transfer of conditions not separated between blocks, the process of transferring conditions is single-step.

In contrast to [18], in which two-step difference equations were studied, here the processes are described by three-step difference equations. It is clear that these systems and the corresponding problems are reducible to each other (i.e., second-order equations can be reduced to a system of first-order equations and vice versa), but the resulting calculation formulas for solving boundary value problems are most effective, as is known, in those cases when they are obtained in terms of the set problem, as many researchers recommend [12, 13, 19] to solve the problem directly in a given formulation, without reducing them to any other equivalent form. This also facilitates the interpretation of the results of the solution, leads to a significant reduction in time and loss of accuracy in the transition from one variable to another, which is associated with a certain additional amount of computational work required.

2. Problem statement

We consider the problem of calculating the state of a complex discrete process consisting of L mutually independent discrete subprocesses described by tridiagonal linear algebraic equations:

$$A_i^k y_{i+1}^k - C_i^k y_i^k + B_i^k y_{i-1}^k = -F_i^k, \qquad k = 1,\dots,L, \qquad i = 1,\dots,n_k - 1. \tag{1}$$

The subprocesses are interconnected by means of $m = 2L$ initial and final states in the form of boundary conditions not separated between blocks, given in the form:

$$\hat{G}\ \ y_n + \check{G} y_{n-1} + \hat{Q} y_1 + \check{Q} y_0 = r. \tag{2}$$

Here, $y^k = \left(y_0^k,\dots,y_{n_k}^k\right)^* \in R^{n_k}$ is a n_k-dimensional vector determining the state of the k-th process; $A_i^k, B_i^k, C_i^k \neq 0$ and F_i^k are prescribed numbers; n_k is the duration of the k- th process, $k = 1,\dots,L$; $(y_0, y_1) = ((y_0^1,\dots,y_0^L)^*, (y_1^1,\dots,y_1^L)^*) \in R^{\mathrm{m}}$, and $(y_{n-1}, y_n) = ((y_{n_1-1}^1,\dots,y_{n_L-1}^L)^*,$ $\left(y_{n_1}^1,\dots,y_{n_L}^L)^*\right) \in R^{\mathrm{m}}$, respectively, states of all subprocesses at the initial and final (individual for each subprocess) time points; $G = (\hat{G}, \check{G})$ и $Q = (\hat{Q}, \check{Q})$ are prescribed $m \times m$ matrices, $r = (r^1,\dots,r^m)^*$ is a prescribed m-dimensional vector, * is the transposition sign. We assume that the rank of the extended matrix (G, Q), i.e. $rang\ (G, Q) = m$, and system of equations (1), (2) on the whole has a solution, and a unique one at that.

Introducting the notation

$$\hat{G} = \left(\hat{g}^{1s},\dots,\hat{g}^{Ls}\right), \qquad \check{G} = \left(\check{g}^{1s},\dots,\check{g}^{Ls}\right),$$

$$\hat{Q} = \left(\hat{q}^{1s},\dots,\hat{q}^{Ls}\right),\ \ \check{Q} = \left(\check{q}^{1s},\dots,\check{q}^{Ls}\right), \qquad s = 1,\dots,m,$$

we write conditions (2) in vector form:

$$\sum_{j=1}^{L}\left[\hat{g}^{sj}y_{n_j}^{j}+\breve{g}^{sj}y_{n_j-1}^{j}\right]+\sum_{j=1}^{L}\left[\hat{q}^{sj}y_{1}^{j}+\breve{q}^{sj}y_{0}^{j}\right]=r^{s},\qquad s=\overline{1,m}, \tag{3}$$

Condition (2), in particular, can take the form:

$$\sum_{j=1}^{L}\left[\hat{q}^{sj}y_{1}^{j}+\breve{q}^{sj}y_{0}^{j}\right]=r^{s},\qquad s=\overline{1,L_1}. \tag{4}$$

$$\sum_{j=1}^{L}\left[\hat{g}^{sj}y_{n_j}^{j}+\breve{g}^{sj}y_{n_j-1}^{j}\right]=r^{s},\qquad s=\overline{1,L_2}. \tag{5}$$

where $L_1+L_2=m$.

The mathematical model of the process under consideration is characterized by: a large number of subprocesses L; a high dimensionality of the state vector of subprocesses or a long duration of their functioning n_k, $k=1,\dots,L$; weak and arbitrary relationships between subprocesses, i.e. weak and arbitrary filling of the matrices $G,\ Q$.

The first two features for real processes lead to the fact that the order of the algebraic system (1), (3) equal to M ($M=N+m-$ is the number of unknowns in system (1) consisting of L subsystems (blocks) with a total number of $N=\sum_{k=1}^{L}n_k-L$ equations), can exceed several thousand and tens of thousands. The third feature leads to non-separated boundary conditions, which necessitates the use of methods for transferring boundary conditions.

The most effective numerical methods for solving systems of high dimensionality N require the N^2 order of arithmetic operations and their direct solution, when N takes on the value of tens of thousands, it requires a lot of processor time even with modern computers, especially when it is necessary to repeatedly solve similar problems, for instance, during optimization. It is clear that the effectiveness of the methods used for solving subtasks, calculating the state of processes, largely affects the efficiency of solving the main problems of optimization and optimal control of their functioning. In particular, applying grid methods [7–9] to solve problems regarding systems of second-order differential equations with partial derivatives of parabolic and hyperbolic type with nonlocal conditions, we obtain systems of the form (1), (2). And the effectiveness of solving the problem of optimal control of an object with distributed parameters will substantially depend on the effectiveness of solving this system.

3. Numerical solution of the problem

We propose an approach to solving the problem under consideration. It is based on the developed algorithm for block transfer of boundary conditions (3) to one end: to the left or to the right. This means that replacing relations (4) or (5) with equivalent relations, for instance, when transferring conditions to the right end, we obtain:

$$\bar{\bar{G}}y_n+\bar{\bar{\breve{G}}}y_{n-1}=\bar{r}, \tag{6}$$

or

$$\sum_{j=1}^{L}\left[\bar{\bar{g}}^{sj}y_{n_j}^{j}+\bar{\bar{\breve{g}}}^{sj}y_{n_j-1}^{j}\right]=\bar{r}^{s},\qquad s=1,\dots,m, \tag{7}$$

in which the variables y_0, y_1 are not involved. When transferring the conditions to the left end, we get:

$$\bar{\bar{Q}}y_1+\bar{\bar{\breve{Q}}}y_0=\bar{r}, \tag{8}$$

or

$$\sum_{j=1}^{L}\left[\bar{\hat{q}}^{sj}y_1^j+\bar{\check{q}}^{sj}y_0^j\right]=\bar{r}^s,\qquad s=1,\dots,m, \tag{9}$$

in which the variables$y_n,\ y_{n-1}$ are not involved. When transferring the conditions to one end, we get system (6), (7) or (8), (9). Solving the resulting system of m algebraic equations with m unknowns, we obtain the values of $y_0,\ y_1$ or $y_n,\ y_{n-1}$, respectively. Further, making simple direct calculations using explicit inverse recurrence formulas with respect to the sweep coefficients, we obtain a solution to the whole problem (1), (2).

We transfer conditions (2) or (3) in the direction in which one of the matrices G and Q is less filled. For instance, if the matrix G is less filled than Q, then conditions (2) or (3) must be transferred to the left, otherwise the conditions should be transferred to the right. After the description of the transfer algorithm is given, this indication will become clearer.

We shall transfer each s-th condition from (2) or from (3) separately: block by block and step by step, $s=1,\dots,m$..

Thus, we consider an arbitrary s-th condition from (3), which after its transfer to the right will take the form of (7). We obtain the conditions in the form (7) step by step. After one step of the transfer scheme, i.e. after the transfer of the s-th condition from (3) to the right, we obtain the condition in the form of (7), or in the form of (9) after the transfer of the s-th condition to the left.

Suppose that among the coefficients q̂_ ^ sj, ^ q̌〗 _ ^ sj, j = 1, ..., L, there are nonzero ones; otherwise, the s-th condition should not be transferred to the right, since only the variables y_n, y_(n-1) are involved in this condition. Suppose the first non-zero coefficient is q̂_ ^ sk, 〖q̌〗 _ ^ sk, i.e. q̂_ ^ sk ≠ 0 and / or q̌_ ^ sk ≠ 0, q̂_ ^ sj = 0, q̌_ ^ sj = 0, j <k.

Suppose that there are nonzero coefficients among the coefficients $\hat{q}^{sj},\ \check{q}^{sj}, j=1,\dots,L$, otherwise, the s-th condition should not be transferred to the right, since only the variables y_n, y_{n-1} are involved in this condition. Suppose the first non-zero coefficient is $\hat{q}^{sk},\ \check{q}^{sk}$, i.e. $\hat{q}^{sk}\neq 0$ and/or $\check{q}^{sk}\neq 0,\ \ \hat{q}^{sj}=0,\ \ \check{q}^{sj}=0,\ \ j<k$.

We introduce the coefficients $\alpha_i^{sk},\ \ \beta_i^{sk},\ \ \gamma_i^{sk}$ such that for $i=0,\dots,n_k-1$, $s=1,\dots,m$, the following relation holds, which is equivalent to (4):

$$\alpha_i^{sk}y_i^k+\beta_i^{sk}y_{i+1}^k+\sum_{j=k+1}^{L}\hat{q}^{sj}y_1^j+\sum_{j=k+1}^{L}\check{q}^{sj}y_0^j+\sum_{j=1}^{L}\hat{g}^{sj}y_{n_j}^j+\sum_{j=1}^{L}\check{g}^{sj}y_{n_j-1}^j=\gamma_i^{sk}, \tag{10}$$

here,

$$\alpha_0^{sk}=\check{q}^{sk},\qquad \beta_0^{sk}=\hat{q}^{sk},\qquad \gamma_0^{sk}=r^s. \tag{11}$$

The coefficients themselves shall be called sweep coefficients.

Definition 1. We shall say that the variables $\alpha_i^{sk},\ \beta_i^{sk},\ \gamma_i^{sk},\ \ i=1,\dots,n_k$ transfer the left value of the solution of the k-th subsystem (1) in the s-th condition (3) to the right if for any solution of k-th subsystem (1) equality (10) holds.

Theorem 1. If the values $\hat{q}^{sk}$ и $\check{q}^{sk}$ at the same time are not equal to zero, then $\alpha_i^{sk},\ \beta_i^{sk},\ \ \gamma_i^{sk}$ determined from the recurrence relations:

$$\alpha_i^{sk} = \beta_{i-1}^{sk} + \alpha_{i-1}^{sk}\frac{C_i^k}{B_i^k}, \qquad \alpha_0^{sk} = \check{q}^{sk},$$
$$\beta_i^{sk} = \left(\beta_{i-1}^{sk} - \alpha_i^{sk}\right)\frac{A_i^k}{C_i^k}, \qquad \beta_0^{sk} = \hat{q}^{sk}, \qquad i = 1,\ldots,n_k, \tag{12}$$
$$\gamma_i^{sk} = \gamma_{i-1}^{sk} + (\alpha_i^{sk} - \beta_{i-1}^{sk})\frac{F_i^k}{C_i^k}, \qquad \gamma_0^{s1} = r^s,$$
$$\gamma_0^{sk+1} = \gamma_{n_k}^{sk},$$

are the right sweep coefficients for the s-th condition (3) with respect to the solution of the k-th subsystem of system of equations (1).

Proof. When $i = 0$, according to (11), condition (10) is equivalent to s-th condition (3). Suppose α_{i-1}^{sk}, β_{i-1}^{sk},γ_{i-1}^{sk} satisfy the condition of sweep of the s-th condition with respect to y^k −solution to k-th equation (1). Consequently, the following takes place:

$$\alpha_{i-1}^{sk}y_{i-1}^k + \beta_{i-1}^{sk}y_i^k + \sum_{j=k+1}^{L}\hat{q}^{sj}y_1^j + \sum_{j=k+1}^{L}\check{q}^{sj}y_0^j + \sum_{k=1}^{L}\hat{g}^{sj}y_{n_k}^j + \sum_{j=1}^{L}\check{g}^{sj}y_{n_j-1}^j = \gamma_{i-1}^{sk}. \tag{13}$$

Subtracting (13) from (10), we get:

$$(\alpha_i^{sk} - \beta_{i-1}^{sk})y_i^k + \beta_i^{sk}y_{i+1}^k - \alpha_{i-1}^{sk}y_{i-1}^k = \gamma_i^{sk} - \gamma_{i-1}^{sk}. \tag{14}$$

We shall write equation (1) in the following explicit form:

$$y_i^k = \frac{A_i^k}{C_i^k}y_{i+1}^k + \frac{B_i^k}{C_i^k}y_{i-1}^k + \frac{F_i^k}{C_i^k}, \quad k = 1,\ldots,L, \quad i = 1,\ldots,n_k - 1. \tag{15}$$

Substituting expression (15) for solving the i-th discrete time instant of the k-th equation in (14), after grouping we get:

$$\left[(\alpha_i^{sk} - \beta_{i-1}^{sk})\frac{A_i^k}{C_i^k} + \beta_i^{sk}\right]y_{i+1}^k + \left[(\alpha_i^{sk} - \beta_{i-1}^{sk})\frac{B_i^k}{C_i^k} - \alpha_{i-1}^{sk}\right]y_{i-1}^k + \left[\left(\alpha_i^{sk} - \beta_{i-1}^{sk}\right)\frac{F_i^k}{C_i^k}\right] =$$
$$= \gamma_i^{sk} - \gamma_{i-1}^{sk}.$$

Considering that this equality must be satisfied for all $i = 1,2,\ldots,n_k$, for all solutions of the k-th subsystem of equations (1), we shall require that α_i^{sk}, β_i^{sk}, γ_i^{sk}, α_{i-1}^{sk}, β_{i-1}^{sk}, γ_{i-1}^{sk} should fulfill the equality to zero of the expressions in square brackets. Hence, we obtain the necessary relations for the sweep coefficients in the form of (12).

Substituting the values of the running coefficients for $i = n_k$ in (10), we obtain a new condition:

$$\sum_{j=1}^{L}\tilde{\hat{g}}^{sj}y_{n_j}^j + \sum_{j=1}^{L}\tilde{\check{g}}^{sj}y_{n_j-1}^j = \gamma_{n_L}^{sL}, \quad s = \overline{1,m}, \tag{16}$$

where $\tilde{\check{g}}^{sj} = \check{g}^{sj} + \alpha_{n_j}^{sj}$, $\tilde{\hat{g}}^{sj} = \hat{g}^{sj} + \beta_{n_j}^{sj}$, $j = 1,\ldots,L$. Performing these operations for all conditions (3) for $s = 1,\ldots,m$, in the end we get an algebraic system of equations in which all values will be given on the right end. Solving system (16), for instance, by the Gaussian elimination, we obtain values for $y_{n_j}^j$, $y_{n_j-1}^j$, $j = 1,\ldots,L$. With this, the forward process of the right sweep ends. To reverse the sweep method for $k = L$, substituting the obtained values $y_{n_j}^j$, $y_{n_j-1}^j$, $j = 1,\ldots,L$ in (10) for any s-th condition, we obtain

$$\alpha_i^{sL} y_i^L + \beta_i^{sL} y_{i+1}^L + \sum_{j=1}^{L} \hat{g}^{sj} y_{n_j}^j + \sum_{j=1}^{L} \breve{g}^{sj} y_{n_j-1}^j = \gamma_i^{sL}.$$

We denote

$$\tilde{r}^L = \sum\nolimits_{j=1}^{L} \left(\hat{g}^{sj} + \beta_{n_j}^{sj}\right) y_{n_j}^j + \sum\nolimits_{j=1}^{L} \left(\breve{g}^{sj} + \alpha_{n_j}^{sj}\right) y_{n_j-1.}^j$$

To calculate all values of $y^k = \left(y_0^k, \ldots, y_{n_k}^k\right)$, $k = L, \ldots, 1$, we shall use the following recurrence formulas for the reverse sweep method:

$$y_i^k = \frac{\gamma_i^{sk} - \tilde{r}^k}{\alpha_i^{sk}} - \frac{\beta_i^{sk}}{\alpha_i^{sk}} y_{i+1}^k, \qquad k = L, \ldots, 1, \quad i = n_k - 2, \ldots, 0.$$
$$\tilde{r}^{k-1} = \tilde{r}^k + \hat{q}^{sk} y_1^k + \breve{q}^{sk} y_0^k,$$

With this, the reverse process of the method ends.

Similarly to the above procedure of transferring conditions to the right end in order to obtain conditions (8) or (9) equivalent to conditions (4), the conditions are sequentially transferred to the left end.

Suppose that there are nonzero coefficients among the coefficients $\hat{g}^{sj}$, $\breve{g}^{sj}$, $j = 1, \ldots, L$, otherwise, the s-th condition should not be transferred to the right, since only the variables y_0, y_1 are involved in this condition. Suppose the first non-zero coefficient is $\hat{g}^{sk}$, $\breve{g}^{sk}$, i.e. $\hat{g}^{sk} \neq 0$ and/or $\breve{g}^{sk} \neq 0$, $\hat{g}^{sj} = 0$, $\breve{g}^{sj} = 0$, $j < k$.

We introduce the coefficients α_i^{sk}, β_i^{sk}, γ_i^{sk} such that for $i = n_k - 1, \ldots, 0$, $s = 1, \ldots, m,$ the following relation holds, which is equivalent to (4):

$$\alpha_i^{sk} y_i^k + \beta_i^{sk} y_{i+1}^k + \sum_{j=1}^{L} \hat{q}^{sj} y_1^j + \sum_{j=1}^{L} \breve{q}^{sj} y_0^j + \sum_{j=k+1}^{L} \hat{g}^{sj} y_{n_j}^j + \sum_{j=k+1}^{L} \breve{g}^{sj} y_{n_j-1}^j = \gamma_i^{sk}, \tag{17}$$

asuming that,

$$\alpha_{n_k}^{sk} = \breve{g}^{sk}, \ \beta_{n_k}^{sk} = \hat{g}^{sk}, \ \gamma_{n_k}^{sk} = r^s.$$

Definition 2. We shall say that the variablesα_i^{sk}, β_i^{sk}, γ_i^{sk}, $i = 1, \ldots, n_k$ transfer the right value of the solution of k-th subsystem (1) in s-th condition (3) to the left if for any solution of k-th subsystem (1) equalities (17) hold.

Theorem 2. If the values $\hat{q}^{sk}$ и $\breve{q}^{sk}$ at the same time are not equal to zero, then the variables α_i^{sk}, β_i^{sk}, γ_i^{sk}, $i = 1, \ldots, n_k$ determined from the recurrence relations:

$$\alpha_i^{sk} = \beta_{i+1}^{sk} + \alpha_{i+1}^{sk} \frac{C_i^k}{B_i^k}, \qquad \alpha_{n_k}^{sk} = \breve{g}^{sk},$$

$$\beta_i^{sk} = (\beta_{i+1}^{sk} - \alpha_i^{sk}) \frac{A_i^k}{C_i^k}, \qquad \beta_{n_k}^{sk} = \hat{g}^{sk},$$

$$\gamma_i^{sk} = \gamma_{i+1}^{sk} + (\alpha_i^{sk} - \beta_{i+1}^{sk}) \frac{F_i^k}{C_i^k}, \qquad \gamma_{n_\nu}^{s1} = r^s, \qquad i = n_k - 1, \ldots, 0,$$
$$\gamma_{n_\nu}^{sk+1} = \gamma_0^{sk}, \qquad k = 1, \ldots, L-1,$$

are the left sweep coefficients for s-th condition (3) with respect to the solution of the k-th subsystem of system of equations (1).

The proof of the theorem is similar to the above proof of Theorem 1.

Substituting the values of the running coefficients for $i = 0$ in (17), we obtain a new condition:

$$\sum_{j=1}^{L} \tilde{\tilde{q}}^{sj} y_1^j + \sum_{j=1}^{L} \tilde{\tilde{q}}^{sj} y_0^j = \gamma_0^{sL}, \quad s = \overline{1,m}, \tag{18}$$

where $\tilde{\tilde{q}}^{sj} = \breve{q}^{sj} + \alpha_0^{sj}$, $\tilde{\tilde{q}}^{sj} = \hat{q}^{sj} + \beta_0^{sj}$, $j = 1,\ldots,L$. Performing these operations for all conditions (3) for $s = 1,\ldots,m$, in the end we get an algebraic system of equations in which all values will be given on the left end. Solving system (18), we obtain values for y_0^j, y_1^j, $j = 1,\ldots,L$. With this, the forward process of the left sweep ends. To reverse the sweep method for $k = L$, substituting the obtained values y_0^j, y_1^j, $j = 1,\ldots,L$ in (17). For any s-th condition, we obtain:

$$\alpha_i^{Ls} y_i^L + \beta_i^L y_{i+1}^L + \sum_{j=1}^{L} \hat{q}^{sj} y_1^j + \sum_{j=1}^{L} \breve{q}^{sj} y_0^j = \gamma_i^{sL}.$$

We denote

$$\bar{r}^L = \sum\nolimits_{j=1}^{L} \left(\hat{q}^{sj} + \beta_0^{sj}\right) y_1^j + \sum\nolimits_{j=1}^{L} \left(\breve{q}^{sj} + \alpha_0^{sj}\right) y_0^j.$$

To calculate all values of $y^k = \left(y_0^k,\ldots,y_{n_k}^k\right)$, $k = L,\ldots,1$, we shall use the following recurrence formulas for the reverse sweep method:

$$\begin{aligned} y_{i+1}^k &= \frac{\gamma_i^{sk} - \bar{r}^k}{\beta_i^{sk}} - \frac{\alpha_i^{sk}}{\beta_i^{sk}} y_i^k, \\ \bar{r}^k &= \bar{r}^{k+1} + \hat{g}^{sk} y_{n_k}^{k+1} + \breve{g}^{sk} y_{n_k-1}^{k+1}, \end{aligned} \qquad k = L,\ldots,1, \quad i = 1,\ldots,n_k - 1.$$

With this, the reverse process of the method ends.

4. Results of numerical experiments.

We shall consider the results of a numerical solution to the following test problem with respect to a discrete process, the state of which is described by a system of tridiagonal algebraic equations:

$$\begin{aligned} 2y_{i+1}^1 - 5y_i^1 + 3y_{i-1}^1 &= -F_i^1, \quad i = 1,\ldots,n-1 \\ 3y_{i+1}^1 - 7y_i^1 + 4y_{i-1}^1 &= -F_i^1, \quad i = 1,\ldots,n-1, \end{aligned} \tag{19}$$

with non-separated conditions given in the following form:

$$\begin{gathered} 0.6y_0^1 + y_1^1 + 2y_0^2 + 2y_1^2 = 0.0597359\,, \\ 0.8y_0^1 + y_1^1 + y_0^2 = 0.0500995, \\ y_n^1 + y_{n-1}^1 + y_n^2 + 2y_{n-1}^2 = 8.1544575, \\ y_n^1 + y_{n-1}^1 + y_n^2 = 10.2054663. \end{gathered} \tag{20}$$

The input data and the results of the calculations performed with double accuracy are given with an accuracy of 10^{-7}. In this problem, $= 100$, $L = 2$. Its exact solution are the functions

$$\begin{aligned} u^1(x_i) &= 2x_i + {x_i}^2 + 3\sin x_i, \qquad x \in [0;1], \\ u^2(x_i) &= ({x_i}^2 + 2x_i)\sin(50x_i), \qquad x \in [0;1]. \end{aligned}$$

To construct the test problem, the segment $x \in [0;1]$ was divided by points $x_i = ih$ into $n = 100$ equal parts, with the step $h = 1/n$. Substituting the values of y_i^k into expressions (19), we calculated the values of F_i^k, $i = 1,\ldots,n-1$, $k = 1,2$.

To solve problem (19), (20) numerically, using the formulas of the right sweep (12), transferring two conditions from the left end to the right, we obtain an agebraic system of the form (6), where

$$\tilde{G}=\begin{pmatrix}4.8 & -3.2\\ 5.4 & -3.6\\ 1 & 1\\ 1 & 1\end{pmatrix},\quad \tilde{Q}=\begin{pmatrix}11.999 & -8.999\\ 3.999 & 2.999\\ 1 & 1\\ 0 & 1\end{pmatrix},\quad \tilde{r}=\begin{pmatrix}-8.959\\ 3.798\\ 8.154\\ 10.205\end{pmatrix}.$$

Solving system of equations (12) by the Gaussian elimination, we obtain the values at the right end of the segment that coincide with the exact values of the functions with an accuracy of 10^{-14}:

$$\begin{aligned} y_n^1 &= 5.4681779, & u^1(1) &= 5.4681779,\\ y_{n-1}^1 &= 5.5244129, & u^1(1-h) &= 5.5244129,\\ y_n^2 &= -2.0510087, & u^2(1) &= -2.0510087,\\ y_{n-1}^2 &= -0.7871245, & u^2(1-h) &= -0.7871245. \end{aligned}$$

The values of the functions obtained using the reverse process of the sweep method coincide with the exact values of the functions with an accuracy of up to 10^{-14} over the entire segment.

Now suppose that the non-separated conditions are given in the following form, similar to (20):

$$2.3y_0^1 + 3y_0^2 + 0.8y_n^1 = 4.4195303,$$

$$1.4y_0^1 + 2y_0^2 + 3y_n^2 = -2.3613736,$$

$$2.4y_0^1 + 1.3y_n^1 + 0.8y_n^2 = 6.5520371,$$

$$0.5y_0^2 + 3y_n^1 + 1.2y_n^2 = 15.6286893.$$

and we need to transfer all the conditions to the right end. Using formulas of the right sweep (12), having transferred all the conditions from the left end to the right, we obtain an algebraic system of the form (6), where

$$(\tilde{G},\tilde{Q})=\begin{pmatrix}4.42 & -2.361 & 6.552 & 15.629\\ 6.9 & -3.6 & 11.999 & -8.999\\ 4.2 & -2.8 & 7.999 & -2.999\\ 7.2 & -3.5 & 0 & 0.8\end{pmatrix},\ \tilde{r}=\begin{pmatrix}-0.790\\ -6.549\\ 19.405\\ 12.707\end{pmatrix}.$$

Solving system of equations (12) by the Gaussian elimination, we obtain the values at the right end of the segment that coincide with the exact values of the functions with an accuracy of up to 10^{-14}. The values of the functions obtained using the reverse process of the sweep method also coincide with an accuracy of up to 10^{-14} with the exact values of the functions over the entire segment.

5. Conclusion

The paper investigates the problem of calculating the state of complex discrete processes described by systems of linear three-step discrete systems of equations of block structure, high dimensionality, with weak and arbitrary relationships between the values of the initial and final variables in subsystems. In particular, the use of finite difference methods in calculating the state and optimizing the parameters of many processes consisting of subprocesses described by systems of equations with ordinary or partial derivatives leads to the solution of such systems. Schemes and corresponding formulas based on the developed block method of transferring conditions that take into account the specifics of the Jacobian of the system and the weakness of the Jacobian matrix of the conditions of relationships between the subsystems are proposed.

The results of numerical experiments obtained in solving the test problem are presented.

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