A method for describing the terrain to determine the effective flight path of a drone

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ABSTRACT

Depending on the task of terrain clearance by autonomous drones, the problem arises of calculating the flight path based on data on stationary obstacles related to individual buildings, on the terrain, etc. The necessary information about the area is usually extracted from topographic maps online, which requires additional technical resources (primarily sufficient RAM) and processing time. The authors propose to use the description of obstacles and terrain features in place of the aforementioned maps with a very limited amount of parametric data representing them in the form of simple geometric figures (cylinder, sphere, etc.). Such a description of the area is proposed to be called a geometric map. An example of an algorithm that allows quickly calculating the optimal path for avoiding an obstacle is presented.

1. Introduction

The rapid development of modern technology has filled the market of high-tech products with various unmanned aerial vehicles (drones), including autonomous, independently controlled ones. Depending on the technical capabilities and functionality, in order to perform certain tasks with autonomous drones, one often has to determine the flight path based on the data on stationary obstacles in the form of individual large objects (tall trees, buildings, water towers, towers, bridges, other structures), on the terrain, etc. This task is especially relevant for flights at low altitudes near populated areas.

In [1], when studying a dynamic drone model, it was assumed that there are no obstacles in the drone's flight space. However, in real flight conditions, it becomes necessary to take into account the terrain and existing obstacles. When determining the flight path using maps of traditional format, some preliminary processing is required, which complicates online calculations. Therefore, the need to develop maps of a format that would be more suitable for these purposes is relevant.

2. Problem statement

When designing the flight path of an aerial vehicle at low altitudes, it is necessary to take into

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account the possibility of flying around obstacles or flying over elevations. To do this, one must first have information about the safe minimum height in certain areas, as well as the presence of elevations in the form of towers, tall buildings, tall trees, etc., which are obstacles along the route. The listed types of obstacles can be easily described by a limited set of data, which will greatly facilitate their accounting during calculations. Therefore, they can be described with some accuracy due to a safe distance as regular geometric shapes (cylinder, sphere, part of a plane, parallelepiped) and such a representation of the terrain can be called a *geometrical map*.

3. Mathematical formalization

Although the generally accepted system for indicating the coordinates of objects on the ground is geographical latitude and longitude, rectangular coordinates will be used in our calculations. Within the flight zone of autonomous aerial vehicles, if necessary, real geographical coordinates with a certain degree of accuracy can be expressed in Cartesian coordinates in some rectangular coordinate system [2,3].

Suppose some rectangular coordinate system, $Oxyz$, is introduced, where the axes $Oxyz$ and Oy are directed in such a way that they form the right system, parallel to the earth's surface here, the Ox axis is directed to the north, and the Oz axis is directed vertically upward.

A geographical map of the area can be composed of the following elements.

Horizontal section – a limited section of the horizontal plane. If we denote by $A_k^{(1)}$, $A_k^{(2)}$, ... , $A_k^{(m_k)}$, the vertices of that m_k-gon $P_k\left[A_k^{(1)}, A_k^{(2)}, \dots, A_k^{(m_k)}\right]$, which are projections of sections on the Oxy plane, they can be written in the form

$$\left\{z_k^h, (x,y) \in P_k\left[A_k^{(1)}, A_k^{(2)}, \dots, A_k^{(m_k)}\right], \delta_k^h, k = 1,2,\dots,k_h\right\}.$$

Here, z_k^h is the height of the section relative to the zero level of calculation;, k_h is the total number of such sections in the map under consideration, δ_k^h is the minimum permissible distance of the aerial vehicle approaching that section from above.

In particular, if the horizontal section is a rectangular section with sides parallel to the coordinate axes, then it can be described as follows:

$$\{z_k^h, x_{0,k}^h \le x \le x_{1,k}^h, y_{0,k}^h \le y \le y_{1,k}^h, \delta_k^h, k = 1,2,\dots,k_h\},$$

where $x_{0,k}^h$, $x_{1,k}^h$, $y_{0,k}^h$, $y_{1,k}^h$ are the prescribed given real numbers representing the geometrical dimensions of the k-th section.

It is advisable to list in the geometric map only those horizontal sections of the terrain when its location height z_k^h differs from the selected zero level by more than by a certain value. The mean value of the level of the terrain over which the flight of the aerial vehicle is planned can be selected as the zero level.

Vertical barrier is a part of some plane perpendicular to the horizontal plane of the terrain. It can be set in two ways:

$$\{a_k^v x + b_k^v y = c_k^v, x_{0,k}^v \le x \le x_{1,k}^v, \delta_k^v, k = 1,2,\dots,k_v\} \text{ или}$$

$$\{a_k^v x + b_k^v y = c_k^v, y_{0,k}^v \le y \le y_{1,k}^v, \delta_k^v, k = 1,2,\dots,k_v\}.$$

Here, a_k^v, b_k^v, c_k^v, $x_{0,k}^v$, $x_{1,k}^v$, $y_{0,k}^v$, $y_{1,k}^v$ are the prescribed given real numbers representing the geometrical location of the barrier on the terrain, k_v is the total number of such barriers in the map under consideration, δ_k^v is the minimum permissible distance of the aerial vehicle approaching that barrier.

Vertical wall is a part of a vertical barrier bounded by z, obviously, it can also be set in two ways:

$$\{a_k^v x + b_k^v y = c_k^v, x_{0,k}^v \le x \le x_{1,k}^v,\ z_k^v,\ \delta_k^v, k = 1,2,\dots,k_v\} \text{ или}$$

$$\{a_k^v x + b_k^v y = c_k^v, y_{0,k}^v \le y \le y_{1,k}^v,\ z_k^v,\ \delta_k^v, k = 1,2,\dots,k_v\}.$$

Here, a_k^v, b_k^v, c_k^v, $x_{0,k}^v$, $x_{1,k}^v$, $y_{0,k}^v$, $y_{1,k}^v$ are the prescribed given real numbers representing the geometrical location of the wall on the terrain, z_k^v is the prescribed height of the wall, k_v is the total number of such walls in the map under consideration, δ_k^v is the minimum permissible distance of the aerial vehicle approaching that wall.

Vertical tower is a geometrical object that is a vertical line. The vertical tower is set by coordinates on the plane Oxy:

$$\{(x_k^t, y_k^t), \delta_k^t, k = 1,2,\dots,k_t\},$$

where x_k^t, y_k^t are the prescribed given real numbers, the coordinates of the geometrical location of the object, k_t is the total number of such objects in the map under consideration, δ_k^t s the minimum permissible distance of the aerial vehicle approaching that object.

A spatial point is a geometrical object representing a certain point in space. It is set by its coordinates in space:

$$\{(x_k^s, y_k^s, z_k^s), \delta_k^s, k = 1,2,\dots,k_s\}.$$

Here, x_k^s, y_k^s, z_k^s are the prescribed given real numbers, the coordinates of the geometrical location of the object, k_s is the total number of such objects in the map under consideration, δ_k^s is the minimum permissible distance of the aerial vehicle approaching that “point”.

Obviously, real construction objects (structures, buildings, etc.) can be described by a set of the above objects. For instance, using these elements, we shall describe a tall building in the shape of a rectangular parallelepiped.

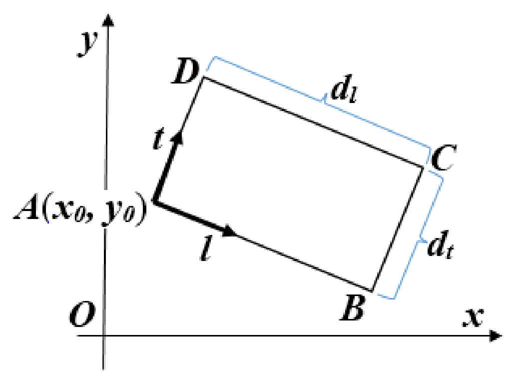

Fig. 1. Parallelogram $P[A, B, C, D]$

We shall denote by $P[A, B, C, D]$ parallelogram with vertices at points A, B, C и D. Let us consider a *parallelepiped* with the base $P[A, B, C, D]$ and the height z_0 located in some point (x_0, y_0) (Fig. 1). Suppose $\boldsymbol{l} = (l_x, l_y)$ is the unit vector of the direction of the longitudinal axis of the projection of the building onto the plane Oxy, d_l and d_t are the length of the projection of the building along the longitudinal and transverse directions, respectively. We shall find the coordinates of the vertices of the parallelogram $P[A, B, C, D]$:

$$\begin{cases} A(x_0, y_0), \\ B(x_0 + d_l l_x, y_0 + d_l l_y), \\ C(x_0 + d_t t_x, y_0 + d_t t_y), \\ D(x_0 + d_l l_x + d_t t_x, y_0 + d_l l_y + d_t t_y). \end{cases}$$

It should be noted that in this context the concepts of "transverse" and "longitudinal" direction are conditional. Now the parallelepiped can be described by the following formulas:

at $l_x \ne 0$

$$\{l_y x - l_x y = l_y x_0 - l_x y_0, min\{x_0, x_0 + d_l l_x\} \leq x \leq max\{x_0, x_0 + d_l l_x\}, z_0, \delta_0\},$$

$$\{l_y x - l_x y = l_y(x_0 + d_t t_x) - l_x(y_0 + d_t t_y), min\{x_0 + d_t t_x, x_0 + d_t t_x + d_l l_x\} \leq x \leq max\{x_0 + d_t t_x, x_0 + d_t t_x + d_l l_x\}, z_0, \delta_0\},$$

$$\{t_y x - t_x y = t_y x_0 - t_x y_0, min\{x_0, x_0 + d_t t_y\} \leq x \leq max\{x_0, x_0 + d_t t_y\}, z_0, \delta_0\},$$

$$\{t_y x - t_x y = t_y(x_0 + d_l l_x) - t_x(y_0 + d_l l_y), min\{x_0 + d_l l_x, x_0 + d_l l_x + d_t t_x\} \leq x \leq max\{x_0 + d_l l_x, x_0 + d_l l_x + d_t t_x\}, z_0, \delta_0\},$$

where $\boldsymbol{t} \equiv (t_x, t_y) = \left(-\frac{l_y}{l_x}, 1\right)$;

at $l_x = 0$

$$\{x = x_0, min\{y_0, y_0 + d_t\} \leq x \leq max\{y_0, y_0 + d_t\}, z_0, \delta_0\},$$

$$\{x = x_0 - d_l, min\{y_0, y_0 + d_t\} \leq x \leq max\{y_0, y_0 + d_t\}, z_0, \delta_0\},$$

$$\{y = y_0, min\{x_0 - d_l, x_0\} \leq x \leq max\{x_0 - d_l, x_0\}, z_0, \delta_0\},$$

$$\{y = y_0 + d_t, min\{x_0 - d_l, x_0\} \leq x \leq max\{x_0 - d_l, x_0\}, z_0, \delta_0\}.$$

Here, δ_0 is the minimum permissible distance of the aerial vehicle approaching this "parallelepiped" (e.g. a bulding). It is clear that the “parallelepiped” object consists of four vertical walls representing the walls of a real building on the ground.

4. An example of using a geometrical map to determine the flight path

Suppose that between the two points of the rectilinear part of the path of the planned drone flight there is a television tower, which must be circumvented along a trajectory consisting of a broken line with two switches. We denote this rectilinear section of the trajectory by $\mathcal{L}$. Moreover, it is required that the total length of the trajectory be minimal. At its core, this is the task of bypassing a vertical tower set in the form $\{O(x_0, y_0), R\}$. The condition for the necessity of determining the calculated trajectory in this case is written as the inequality $\rho(\mathcal{L}, O) < R$, where ρ is the Euclidean distance. For simplicity, we assume that the flight altitude of the aerial vehicle does not change. Then the problem of finding the optimal calculated path can be considered as a geometrical problem on the plane:

– It is required to find a broken line with two breakpoints that connects the starting point $M(x_1, y_1)$ with the end point $N(x_2, y_2)$, bypassing a circle of radius R centered at $O(x_0, y_0)O(x_0, y_0)$ so that its length is minimal.

It is believed that the coordinates of the points M and N are determined depending on the flight speed and maneuvering characteristics of the drone, and they are known.

It is known that from a given point two tangent lines to a given circle can be drawn. Each of these tangent lines defines one of two possible trajectories. One of these trajectories, which is a broken line with two break points, is the solution to the problem. Fig. 2 shows the diagram of the search for the minimum trajectory. Let us find out which of the broken lines D_1E_1N and MD_2E_2N has a minimum length.

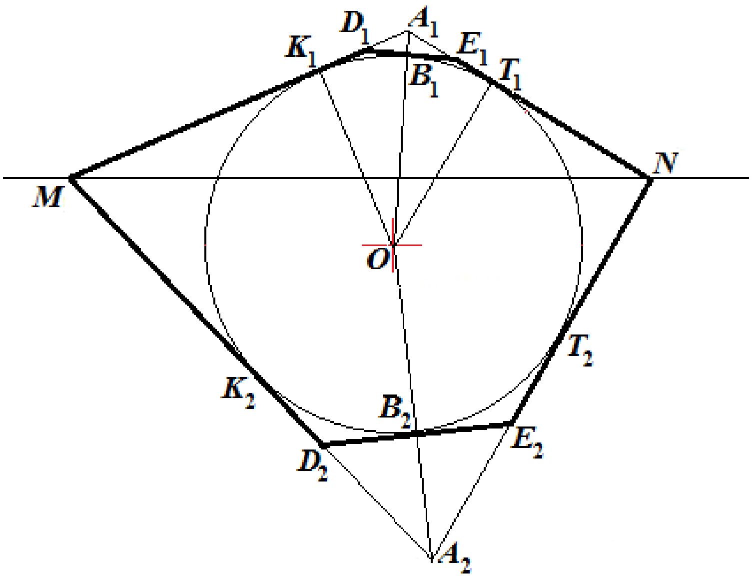

Fig. 2. The minimum trajectory search diagram

We shall give an algorithm for solving the problem. The coordinates of the points $K_i(x_{Ki}, y_{Ki})$, $T_i(x_{Ti}, y_{Ti})$ and $A_i(x_{Ai}, y_{Ai})$ can be calculated respectively from the system of equations (1), (2) and (3), which are a formalization of statements "$MK_i \perp K_iO$", "$|K_iO| = R$", "$NT_i \perp T_iO$", "$|T_iO| = R$", "$A_i = MA_i \cap NA_i$":

$$\begin{cases}(x_M - x_{Ki})(x_O - x_{Ki}) + (y_M - y_{Ki})(y_O - y_{Ki}) = 0, \\ (x_O - x_{Ki})^2 + (y_O - y_{Ki})^2 = R^2,\end{cases} \tag{1}$$

$$\begin{cases}(x_N - x_{Ti})(x_O - x_{Ti}) + (y_N - y_{Ti})(y_O - y_{Ti}) = 0, \\ (x_O - x_{Ti})^2 + (y_O - y_{Ti})^2 = R^2,\end{cases} \tag{2}$$

$$\begin{cases}\frac{x_{Ai} - x_M}{x_{Ki} - x_M} = \frac{y_{Ai} - y_M}{y_{Ki} - y_M}, \\ \frac{x_{Ai} - x_N}{x_{Ti} - x_N} = \frac{y_{Ai} - y_N}{y_{Ti} - y_N}.\end{cases} \tag{3}$$

Since the lengths of the segments tangent to the circle are equal $MK_1 = MK_2$, $NT_1 = NT_2$, then the optimality of the trajectory depends on the length of the segments D_1E_1, D_2E_2. It is easy to see that of these segments, the minimum length is that which is located on the side from the center O where the segment MN itself is located. According to [4, p.117], let us introduce the function

$$f(x, y) = (y_N - y_M)x + (x_M - x_N)y + [(x_M - x_N)y_M - (y_N - y_M)x_M].$$

Then the inequality $f(x_O, y_O) \times f(x_{Ai}, y_{Ai}) > 0$ will mean that the point A_i located in the same side of the center O, where the segment MN is laid. Further, for this index i the coordinates of the points $B_i(x_{Bi}, y_{Bi})$, $D_i(x_{Di}, y_{Di})$ and $E_i(x_{Ei}, y_{Ei})$ can be calculated, respectively, from the system of equations (4), (5) and (6), which are a formalization of statements "the differences $x_{Bi} - x_O$ and $y_{Bi} - y_O$ are proportional respectively to the differences $x_{Ai} - x_O$ and $y_{Ai} - y_O$ with the proportionality coefficient $R/|A_iO|$", "$D_iE_i \perp OB_i$", "$D_i = MA_i \cap D_iE_i$", "$E_i = NA_i \cap D_iE_i$":

$$\frac{x_{Bi} - x_O}{x_{Ai} - x_O} = \frac{y_{Bi} - y_O}{y_{Ai} - y_O} = \frac{R}{\sqrt{(x_{Ai} - x_O)^2 + (y_{Ai} - y_O)^2}}, \tag{4}$$

$$\begin{cases} \frac{x_{Di}-x_{Bi}}{x_{Bi}-x_O} = -\frac{y_{Di}-y_{Bi}}{y_{Bi}-y_O}, \\ \frac{x_{Di}-x_M}{x_{Ki}-x_M} = \frac{y_{Di}-y_M}{y_{Ki}-y_M}, \end{cases} \quad (5)$$

$$\begin{cases} \frac{x_{Ei}-x_{Bi}}{x_{Bi}-x_O} = -\frac{y_{Ei}-y_{Bi}}{y_{Bi}-y_O}, \\ \frac{x_{Ei}-x_N}{x_{Ti}-x_N} = \frac{y_{Ei}-y_N}{y_{Ti}-y_N}. \end{cases} \quad (6)$$

Thus, when planning the flight of an aerial vehicle, a geometrical map allows quickly determining the calculated flight path, taking into account all possible obstacles and features of the terrain.

5. Conclusion

The paper proposes a new method for describing the terrain, which is called a geometrical map. A geometrical map of terrain can be used to effectively determine the flight path of a low-flying vehicle. Representing a geometrical model of the area, it is described as a set of regular geometric shapes. Such a map is relatively easily to store in the "limited memory" of the aerial vehicle. At the same time, the a geometrical map allows quickly calculating the optimal trajectory using the minimum number of values, which are the basic parameters of simple shapes. Such maps can be compiled both by experts and generated on the basis of digital topographic maps of a traditional format (ArcView, DTED, ESRI, etc.) by software specially developed for this purpose. The proposed maps can be used in navigation systems using visual data obtained from cameras located on autonomous drones. Such systems are used in the absence of GPS navigation. Many studies have been devoted to this problem, including [5–13]. Using the proposed maps in these systems will significantly reduce the load on the hardware of drones due to low consumption of RAM and processor resources.

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