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Technology for vibration noise monitoring of the technical condition of railroad tracks

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ABSTRACT

It is shown that modern geometry cars, flaw detector cars and other track test cars provide reliable control of the technical condition of all hauls of the track at "certain intervals of time". Their number is limited and therefore "continuous monitoring" of all stages is almost impossible. One of the possible options for monitoring the technical condition of the track in these "intervals of time" is considered, and while analyzing the soil vibrations arising from the impact of the rolling stock, informative attributes are formed. The application of traditional technologies of correlation and spectral analysis for this purpose proved ineffective, since significant errors arise from the effects of the noise of vibration signals, and the adequacy of the results of track control decreases. For this reason, the authors use the technology of noise analysis of the useful vibration signal, the noise of vibration signals, and the relationship between them. Noise here is used as a carrier of diagnostic information. On their basis, one of the possible options for a technical monitoring tool is proposed, which can be used in one of the cars of all rolling stocks to monitor the beginning of changes in the technical condition of the track during their movement in all hauls.

1. Introduction

It is known that one of the main requirements for a railroad track is that all its elements, track superstructure, artificial structures, as well as the roadbed, must have adequate strength, stability and condition to ensure safe and smooth movement of trains at the speeds established for this particular section [1].

To ensure the safety of the track, it is necessary to obtain sufficient information to monitor the technical condition of the track ballast, the subgrade, under the ballast and sloping areas of the roadbed during the movement of the rolling stock. Due to the importance of this problem, special track test cars are used on railroads. These are geometry cars, flaw detector cars, carriages for

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checking and testing the overhead line [2].

The monitoring of the condition of the track superstructure is mainly carried out by geometry cars designed for continuous high-speed monitoring of the condition of the rail track under dynamic load. Modern geometry cars are complex computerized systems that allow assessing the state of the track by a variety of parameters. They have a standard trolley on one end, and a special one on the other. It contains special equipment that makes all the necessary measurements, for instance, track width, twists, depressions and many other parameters. All these data are recorded on special media by recording devices. Further, after passing a section, all the necessary information about the revealed flaws enters the track maintenance department, with the aim of eliminating them as soon as possible [1, 2]. It operates both at reduced and at maximum speeds established for a given haul. Railroad workers also use flaw detector cars. A flaw detector car is an ordinary car with the necessary equipment and a special trolley, for conducting flaw detection of rails and track switches. In recent years, self-propelled flaw detectors [3-5] equipped with radio communications have been actively introduced to transmit information about detected violations to the duty officers at stations on both ends of a haul, and of course, all the information is sent to the track maintenance department. There is a roadbed diagnostics system, which is based on the use of both traditional and new geophysical methods of modern measuring equipment and computer technology. GPR diagnostics can be implemented in high-speed mode when monitoring a railroad track by means of a geo flaw detector.

Despite all this, due to the importance of the problem of railroad safety, the development of fundamentally new intelligent technologies to obtain information on the condition of the railroad track in an amount sufficient to monitor the beginning of the latent period of changes in its technical condition can be considered a relevant issue.

2. Problem statement

With the development of high-speed train traffic, the requirements for objects and devices of railroad infrastructure are becoming more stringent: both the quality of determining the occupancy of the track and the condition of the rail line, the track superstructure (ballast), on which the qualitative characteristics of performance, safety and uninterrupted operation of trains depend. Therefore, the question of creating new alternative solutions in the field of improving the control of the technical condition of tracks is relevant. In this regard, it is of great practical importance to create new technical solutions in the field of monitoring changes in the technical condition of track structures on in real time during rolling stock movement [1-5].

Currently, in railroad control systems, most measuring mechanisms with sensors are located below the car body for the convenience of monitoring and maintenance. For instance, geometry cars use typical linear displacement sensors, which provide such information as linear vibrational displacements along the coordinate axes, bouncing, lateral, twitching, angular vibrational displacements relative to the coordinate axes, pitching, side rolling [1-5].

The problem this paper aims to solve is the creation of new technologies for monitoring the technical condition of the railway track, which, by analyzing vibration noisy signals, allow revealing its pre-failure states in real time without limitations of the speed of trains.

However, the technical condition is monitored by geometry cars of each haul on schedule, i.e. "in turn", and it is believed that no significant changes occur between the checks. At the same time, in real life, even a day after control, certain changes can take place. Therefore, in addition to the existing ones, we need to have simple and inexpensive monitoring systems, which can be installed on two or three cars of all trains as a device for signaling the beginning of changes in all the hauls along the route of the rolling stock. Due to this, the information center can receive in real time signals from the hauls that need to be monitored out of turn.

It is advisable to take into account that one of the most effective diagnostic methods is based

on the use of vibrations of the soil of embankments caused by the rolling stock [1-5]. The prerequisite for the application of the vibration method is that a certain condition of embankments corresponds to a group of diagnostic signs of a dynamic process that occur during the movement of trains.

Suppose that during the movement of rolling stock at the output of the vibration sensor D_V installed on the car body, the noisy sampled vibration signal $(i\Delta t)$ is obtained, which is caused by the impact of processes related to the technical condition of the car and the mode of movement of the rolling stock. They are due to the effects of linear and angular vibrational displacements, lateral motion, pitching, rolling, etc. In addition, they also reflect the impact of the technical condition of the railroad track. As a result, the noisy vibration signal $(i\Delta t)$ is formed, which consists of the useful vibration signal $(i\Delta t)$ and the sum noise $\varepsilon(i\Delta t)$ of the vibration signal, i.e.:

$$\mathrm{g}(i\Delta t) = \mathrm{X}(i\Delta t) + \varepsilon(i\Delta t)$$

It can be assumed that due to the impact of the technical condition of the track, due to the enormous weight of the car and rolling stock, low-frequency vibrations occur. At the same time, it can also be assumed that high-frequency components are mainly caused by the other above factors. Therefore, it can be assumed that in the sum noisy vibration signal, the high-frequency components $\varepsilon(i\Delta t)$ reflect the technical condition of the car of the rolling stock, and the useful signal consisting of low-frequency components $\mathrm{X}(i\Delta t)$ and the relationship coefficient between them $R_{X\varepsilon}(\mu)$ reflect the information about the technical condition of the track. Therefore, by analyzing the useful vibration signal, the sum noise of the vibration signal and the relationship between them, it is possible to monitor the onset of changes in the technical condition of the track in any haul during the movement of the rolling stock.

In view of the above, in order to monitor the beginning of changes in the technical condition of railroad track hauls during the movement of the rolling stock in real time, it is necessary to create technologies for the analysis of equivalent useful vibration signals and equivalent noise, allowing to obtain results similar to the results of real useful vibration signals and noise, i.e. it is necessary to ensure that the following equalities hold:

$$D_X = \frac{1}{N}\sum_{i=1}^{N} X^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} X^{e^2}(i\Delta t) = D_X^e$$

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon^{e^2}(i\Delta t) = D_\varepsilon^e$$

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon^{e^2}(i\Delta t) = D_\varepsilon^e$$

$$R_{XX}(\mu) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\, X(i+\mu) \approx \frac{1}{N}\sum_{i=1}^{N} X^e(i\Delta t)\, X^e(i+\mu)\Delta t = R_{X^eX^e}^e(\mu)$$

$$R_{\varepsilon\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\, \varepsilon(i+\mu)\Delta t \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon^e(i\Delta t)\, \varepsilon^e(i+\mu)\Delta t = R_{\varepsilon^e\varepsilon^e}^e(\mu)$$

$$R_{X\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\, \varepsilon(i+\mu)\Delta t \approx \frac{1}{N}\sum_{i=1}^{N} X^e(i\Delta t)\, \varepsilon^e(i+\mu)\Delta t = R_{X^e\varepsilon^e}^e(\mu)$$

$$a_{n_X} = \frac{1}{N}\sum_{i=1}^{N} \sin n\,\omega_j X(i\Delta t) \approx a_{n_X}^e \frac{1}{N}\sum_{i=1}^{N} \sin n\,\omega_j X^e(i\Delta t) = a_{n_X}^e$$

$$b_{n_X} = \frac{1}{N}\sum_{i=1}^{N} \cos n\,\omega_j X(i\Delta t) \approx b_{n_X}^{e}\frac{1}{N}\sum_{i=1}^{N} \cos n\,\omega_j X^{e}(i\Delta t) = b_{n_X}^{e}$$

$$a_{n_\varepsilon}^{*} = \frac{1}{N}\sum_{i=1}^{N} sin\, n\,\omega_j \varepsilon(i\Delta t) \approx a_{n_\varepsilon}^{*e}\frac{1}{N}\sum_{i=1}^{N} \sin n\,\omega_j \varepsilon^{e}(i\Delta t) = a_{n_\varepsilon}^{*e}$$

$$b_{n_\varepsilon}^{*} = \frac{1}{N}\sum_{i=1}^{N} \cos n\,\omega_j \varepsilon(i\Delta t) \approx b_{n_\varepsilon}^{*e}\frac{1}{N}\sum_{i=1}^{N} \cos n\,\omega_j \varepsilon^{e}(i\Delta t) = b_{n_\varepsilon}^{*e}$$

where:

– $X(i\Delta t)$, $X^{e}(i\Delta t)$ are the useful vibration signal and the equivalent useful vibration signal, respectively;

– $\varepsilon(i\Delta t)$, $\varepsilon^{e}(i\Delta t)$ are the noise of the vibration signal and the equivalent noise, respectively;

– $R_{XX}(\mu)$, $R_{\varepsilon\varepsilon}(\mu)$, $R_{X^eX^e}(\mu)$, $R_{\varepsilon^e\varepsilon^e}(\mu)$ are the estimates of the correlation functions of the useful signal and the noise, and the estimates of the equivalent correlation functions, the useful signals and the equivalent noises;

– a_{n_X}, b_{n_X}, , a_{n_ε}, b_{n_ε}, $a_{n_X}^{e}$, $b_{n_X}^{e}$, $a_{n_\varepsilon}^{*}$, $b_{n_\varepsilon}^{*}$ are the spectral characteristics of the useful signals and the noises and the estimates of the spectral characteristics of the equivalent useful signals and the equivalent noises, respectively.

3. Difficulties of the application of traditional correlation analysis technologies for monitoring changes in the technical condition of the railroad track

It is known that in the correlation analysis of the vibration signals $X(i\Delta t)$, determining the estimates of the correlation functions $R_{XX}(\mu)$ due to their contamination with the noises $\varepsilon(i\Delta t)$ in practice is associated with serious difficulties [1-5]. This is due to the fact that the real noisy signals $g(i\Delta t)$ received from sensors installed at various objects are the sum of the useful signals $X(i\Delta t)$ and the noises $\varepsilon(i\Delta t)$, i.e.:

$$g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t),$$

where $g(i\Delta t)$, $X(i\Delta t)$, $\varepsilon(i\Delta t)$ are the samples of the centered sampled noisy signal $g(t)$, the useful signal $X(t)$ and the noise $\varepsilon(t)$.

Because of this, when using traditional algorithms to determine the estimates of the correlation functions $R_{gg}(\mu)$ of the noisy signals $g(i\Delta t)$

$$R_{gg}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g\big((i+\mu)\Delta t\big) =$$

$$= \frac{1}{N}\sum_{i=1}^{N}(X(i\Delta t) + \varepsilon(i\Delta t)) \times (X\big((i+\mu)\Delta t\big) + \varepsilon\big((i+\mu)\Delta t\big),$$

errors arise that can be determined from the expression:

$$\lambda_{gg}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) + \varepsilon(i\Delta t)X\big((i+\mu)\Delta t\big) + \varepsilon(i\Delta t)\varepsilon((i+\mu)\Delta t)].$$

Due to this, the following obvious inequalities take place between the estimates of the correlation functions $R_{gg}(\mu)$ and the vibration signal $g(i\Delta t)$ and the useful signal $X(i\Delta t)$ [1-3]:

$$R_{XX}(\mu) \neq R_{gg}(\mu).$$

For this reason, in practice, in many cases, it is impossible to ensure the adequacy of the results of solving diagnostic problems using traditional technologies [1-3]. In this regard, it is necessary to create algorithms and technologies for eliminating the effects of these errors on the result of

determining the estimate$R_{gg}(\mu)$ of the correlation functions of the vibrational signals $g(i\Delta t)$.

In [1], it is shown that the sum noise $\varepsilon(i\Delta t)$ of the vibration signal $g(i\Delta t)$ in many cases consists of the noise $\varepsilon_1(i\Delta t)$ caused by external factors and the noise $\varepsilon_2(i\Delta t)$ caused by the initiation of various defects during the operation of the objects of control, i.e.:

$$\varepsilon(i\Delta t) = \varepsilon_1(i\Delta t) + \varepsilon_2(i\Delta t).$$

Suppose that $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ is a sampled stationary random signal with a normal distribution law consisting of the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ with a zero mathematical expectation [4-9].

In this case, the formula for calculating the estimate $R_{gg}(\mu = 0)$, i.e. the estimate of the variance of the noisy signal, takes the form:

$$D_g \approx R_{gg}(0) = \frac{1}{N}\sum_{i=1}^{N} g^2(i\Delta t) = \frac{1}{N}\sum [X(i\Delta t) + \varepsilon(i\Delta t)][X(i\Delta t) + \varepsilon(i\Delta t)] \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} X^2(i\Delta t) + 2\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t) + \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon(i\Delta t) \approx$$

$$\approx R_{XX}(0) + 2R_{X\varepsilon}(0) + R_{\varepsilon\varepsilon}(0). \quad (1)$$

Therefore, the error of the obtained result will be:

$$\lambda_{gg}(\mu = 0) = 2R_{X\varepsilon}(0) + R_{\varepsilon\varepsilon}(0) = D_\varepsilon, \quad (2)$$

where $R_{X\varepsilon}(0) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t)$ is the cross-correlation function between the useful signal and the noise; $R_{\varepsilon\varepsilon}(0) \approx \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon(i\Delta t)$.

The formula for calculating the estimate of the correlation function $R_{gg}(\mu)$ at $\mu \neq 0$ can also be represented as follows:

$$R_{gg}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g\big((i+\mu)\Delta t\big) \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} (X(i\Delta t) + \varepsilon(i\Delta t)) \times (X\big((i+\mu)\Delta t\big) + \varepsilon\big((i+\mu)\Delta t\big) \approx$$

$$\approx \frac{1}{N}\sum_{i=1}^{N} [X(i\Delta t)X\big((i+\mu)\Delta t\big) + \varepsilon(i\Delta t)X\big((i+\mu)\Delta t\big) + X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) +$$

$$\varepsilon(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) \approx R_{XX}(\mu) + R_{\varepsilon X}(\mu) + +R_{X\varepsilon}(\mu) + R_{\varepsilon\varepsilon}(\mu)\ . \quad (3)$$

Given the obvious equality,

$$R_{X\varepsilon}(\mu) = R_{\varepsilon X}(\mu),$$

$$R_{\varepsilon_i \varepsilon_{i+\mu}}(\mu) = 0,$$

the error $\lambda_{gg}(\mu \neq 0)$ of the obtained result will be:

$$\lambda_{gg}(\mu \neq 0) = 2R_{X\varepsilon}(\mu). \quad (4)$$

Therefore, it is obvious from expressions (1), (3) and (4) that the error in the estimates of traditional technologies for the correlation analysis of noisy vibration signals will be:

$$\lambda_{gg}(\mu) \approx \begin{cases} 2R_{X\varepsilon}(0) + R_{\varepsilon\varepsilon}(0) \ when\ \mu = 0 \\ 2R_{X\varepsilon}(\mu)\ when\ \mu \neq 0 \end{cases} \quad (5)$$

Thus, it is obvious from the above that in order to exclude the error $\lambda_{gg}(\mu)$ from the estimates $R_{gg}(\mu)$ of the results of the correlation analysis of noisy vibration signals obtained by traditional

technologies, it is necessary to create algorithms and technologies for determining the estimates of both the noise variance D_ε and the estimates of the cross-correlation functions between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, i.e. $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, … [1].

4. Algorithms for determining the errors of results of correlation analysis of noisy vibration signals

It follows from equalities (2)-(5) that in order to determine the estimate of the correlation function $R_{XX}(\mu)$ of the useful signal $X(i\Delta t)$, it is necessary to eliminate the errors of traditional algorithms of the correlation analysis of noisy signals.

In [1], it was shown that the noise variance can be determined from the expression:

$$D_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}\left[g^2(i\Delta t) - 2g(i\Delta t)g\big((i+1)\Delta t\big) + g(i\Delta t)g((i+2)\Delta t)\right] \tag{6}$$

The validity of this expression can be verified by expanding its right-hand side into the corresponding terms, i.e.:

$$\begin{aligned} D_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}&\left[g(i\Delta t)g(i\Delta t) - 2g(i\Delta t)g\big((i+1)\Delta t\big) + g(i\Delta t)g\big((i+2)\Delta t\big)\right] \\ &\approx \frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t)+\varepsilon(i\Delta t)][X(i\Delta t)+\varepsilon(i\Delta t)] \\ &-\frac{1}{N}\sum_{i=1}^{N}2[X(i\Delta t)+\varepsilon(i\Delta t)]\big[X\big((i+1)\Delta t\big)+\varepsilon((i+1)\Delta t\big] \\ &+\frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t)+\varepsilon(i\Delta t)][X((i+2)\Delta t)+\varepsilon((i+2)\Delta t)] = \\ &= R_{XX}(0) + R_{X\varepsilon}(0) + R_{\varepsilon X}(0) + R_{\varepsilon\varepsilon}(0) - 2R_{XX}(\Delta t) - 2R_{X\varepsilon}(\Delta t) - R_{\varepsilon X}(\Delta t) - \\ &2R_{\varepsilon\varepsilon}(\Delta t) + R_{XX}(2\Delta t) + R_{X\varepsilon}(2\Delta t) + R_{\varepsilon X}(2\Delta t) + R_{\varepsilon\varepsilon}(2\Delta t). \end{aligned} \tag{7}$$

If the conditions of stationarity and normality of the law of distribution of noisy signals are satisfied here, then the conditions can be considered valid [1, 6-10]:

$$\begin{aligned} &R_{X\varepsilon}(0) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t) \neq 0, \\ &R_{\varepsilon X}(0) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X(i\Delta t) \neq 0, \\ &R_{\varepsilon\varepsilon}(0) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon(i\Delta t) \neq 0, \\ &R_{XX}(0) + R_{XX}(2\Delta t) - 2R_{XX}(\Delta t) \approx 0, \\ &R_{\varepsilon\varepsilon}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon\big((i+1)\Delta t\big) \approx 0, \\ &R_{\varepsilon\varepsilon}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon\big((i+2)\Delta t\big) \approx 0, \\ &R_{X\varepsilon}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{X}(i\Delta t)\varepsilon\big((i+1)\Delta t\big) \approx 0, \\ &R_{X\varepsilon}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{X}(i\Delta t)\varepsilon\big((i+2)\Delta t\big) \approx 0, \\ &R_{\varepsilon X}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\mathrm{X}\big((i+1)\Delta t\big) \approx 0, \\ &R_{\varepsilon X}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\mathrm{X}\big((i+2)\Delta t\big) \approx 0, \end{aligned} \tag{8}$$

Taking into account (8), in the right-hand side of expression (7) we obtain:

$$D_\varepsilon = R_{X\varepsilon}(0) + R_{\varepsilon X}(0) + R_{\varepsilon\varepsilon}(0) \approx 2R_{X\varepsilon}(0) + R_{\varepsilon\varepsilon}(0).$$

Thus, the estimate of the noise variance D_ε, which is determined from formula (6), is the error of the estimate of the correlation function $R_{gg}(\mu)$ at $\mu = 0$. Therefore, the estimate $R_{XX}(\mu = 0)$ of the correlation function of the useful vibration signal $X(i\Delta t)$ contaminated with the noise $\varepsilon(i\Delta t)$ can be determined from the formula

$$R_{XX}(\mu = 0)=R_{gg}(\mu = 0) - D_\varepsilon = R_{gg}(\mu = 0) - [R_{\varepsilon\varepsilon}(\mu = 0) + 2R_{X\varepsilon}(\mu = 0)].$$

It follows from expression (5) that the error of the estimate $R_{gg}(\mu)$ at $\mu \neq 0$ is the estimate of the cross-correlation function $R_{X\varepsilon}(\mu)$ between the useful vibration signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$. Therefore, for this case, the estimate of the correlation function $R_{XX}(\mu)$ of the useful vibration signal $X(i\Delta t)$ at $\mu \neq 0$ can be represented as follows:

$$R_{XX}(\mu\Delta t) = R_{gg}(\mu\Delta t) - 2R_{X\varepsilon}(\mu\Delta t).$$

Therefore, to reduce the error of the correlation analysis of noisy vibration signals at $\mu \neq 0$, we need to determine the estimate $R_{X\varepsilon}(\mu\Delta t)$.

It is shown in [1] that the formula for determining the cross-correlation function between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ at $\mu \neq 0$ can be represented as follows:

$$R'_{X\varepsilon}(\mu = \Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\big[g(i\Delta t)g(i+1) - 2g(i\Delta t)g\big((i+2)\Delta t\big) + g(i\Delta t)g\big((i+3)\Delta t\big)\big].$$

When expanding the right-hand side of this equality, we obtain:

$$\frac{1}{N}\sum_{i=1}^{N}[\,g(i\Delta t)g\big((i+1)\Delta t\big)] - \frac{1}{N}\sum_{i=1}^{N} 2[g(i\Delta t)g((i+2)\Delta t] + \frac{1}{N}\sum_{i=1}^{N}[g(i\Delta t)g((i+3)\Delta t] \approx \frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t) + \varepsilon(i\Delta t)]\big[X\big((i+1)\Delta t\big) + \varepsilon((i+1)\Delta t\big] - \frac{1}{N}\sum_{i=1}^{N} 2[X(i\Delta t) + \varepsilon(i\Delta t)] \times [X((i+2)\Delta t) + \varepsilon((i+2)\Delta t)] + \frac{1}{N}\sum_{i=1}^{N}[X(i\Delta t) + \varepsilon(i\Delta t)]\big[X\big((i+3)\Delta t\big) + \varepsilon\big((i+3)\Delta t\big)\big] \approx R_{XX}(\Delta t) + R_{X\varepsilon}(\Delta t) + R_{\varepsilon X}(\Delta t) + R_{\varepsilon\varepsilon}(\Delta t) - 2R_{XX}(2\Delta t) - 2R_{X\varepsilon}(2\Delta t) - 2R_{\varepsilon X}(2\Delta t) - 2R_{\varepsilon\varepsilon}(2\Delta t) + R_{XX}(3\Delta t) + R_{X\varepsilon}(3\Delta t) + R_{\varepsilon X}(3\Delta t) + R_{\varepsilon\varepsilon}(3\Delta t).$$

Under the conditions of stationarity and normality of the distribution law of noisy vibration signals and in the presence of a correlation between$X(i\Delta t)$ and $\varepsilon\big((i+1)\Delta t\big)$, assuming that the following relationships are true [1, 4-6]

$$R_{X\varepsilon}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon\big((i+1)\Delta t\big) \neq 0,$$
$$R_{\varepsilon X}(\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+1)\Delta t) \neq 0,$$
$$R_{XX}(\Delta t) + R_{XX}(3\Delta t) - 2R_{XX}(2\Delta t) \approx 0,$$
$$R_{\varepsilon\varepsilon}(\Delta t) \approx 0,\ R_{\varepsilon\varepsilon}(3\Delta t) \approx 0,\ R_{\varepsilon\varepsilon}(2\Delta t) \approx 0,$$
$$R_{\varepsilon\varepsilon}(\Delta t) + R_{\varepsilon\varepsilon}(3\Delta t) - 2R_{\varepsilon\varepsilon}(2\Delta t) \approx 0,$$
$$R_{X\varepsilon}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+2)\Delta t) \approx 0,$$
$$R_{X\varepsilon}(3\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon\big((i+3)\Delta t\big) \approx 0,$$
$$R_{\varepsilon X}(2\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+2)\Delta t) \approx 0,$$
$$R_{\varepsilon X}(3\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X\big((i+3)\Delta t\big) \approx 0,$$

we obtain the equality:

$$R'_{X\varepsilon}(\Delta t) \approx R_{X\varepsilon}(\Delta t) + R_{\varepsilon X}(\Delta t) \approx 2R_{X\varepsilon}(\Delta t).$$

Therefore, the estimate of the cross-correlation function$R_{X\varepsilon}(\Delta t)$ between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ at $\mu = \Delta t$ can be calculated from the expression:

$$R_{X\varepsilon}(\Delta t) \approx \frac{R'_{X\varepsilon}(\Delta t)}{2} \approx$$
$$\approx \frac{1}{2N}\sum_{i=1}^{N}\left[g(i\Delta t)g((i+1)\Delta t) - 2g(i\Delta t)g\big((i+2)\Delta t\big) + g(i\Delta t)g\big((i+3)\Delta t\big)\right] \approx$$
$$\approx \frac{1}{2N}\sum_{i=1}^{N}\left[g(i\Delta t)g\big((i+1)\Delta t\big) - 2g(i\Delta t)g\big((i+2)\Delta t\big) + +g(i\Delta t)g\big((i+3)\Delta t\big)\right]. \quad (9)$$

It is clear that $R_{X\varepsilon}(2\Delta t), R_{X\varepsilon}(3\Delta t), \ldots$ can be determined in a similar manner in the presence of a correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ at $\mu = 2\Delta t$, $\mu = 3\Delta t, \ldots$. Therefore, at various time shifts $\mu\Delta t,\ \mu = 1, 2, 3, \ldots$ the estimates $R_{X\varepsilon}(\mu\Delta t)$ can be determined using a similar expression, i.e.:

$$R_{X\varepsilon}(\mu\Delta t) \approx \frac{1}{2N}\sum_{i=1}^{N}[\, g(i\Delta t)g\left(\big(i+(\mu+1)\big)\Delta t\right) -$$
$$-2g(i\Delta t)g\left(\big(i+(\mu+1)\big)\Delta t\right) + g(i\Delta t)g\left(\big(i+(\mu+2)\big)\Delta t\right)]. \quad (10)$$

Thus, the availability of algorithms and technologies for determining the estimates of the noise variance D_ε and cross-correlation function $R_{X\varepsilon}(\mu)$ between the useful signal and the noise opens up the possibility of reducing the error of traditional algorithms for determining the estimates of correlation functions. Due to this, the estimate of the correlation function of the useful vibration signal $X(i\Delta t)$ contaminated with the noise $\varepsilon(i\Delta t)$ can be determined from the expression:

$$R_{XX}(\mu) \approx \begin{cases} R_{gg}(0) - [2R_{X\varepsilon}(0) + R_{\varepsilon\varepsilon}(0)] \ when\ \mu = 0 \\ R_{gg}(\mu) - 2R_{X\varepsilon}(\mu)\ when\ \mu \neq 0 \end{cases},$$

where the estimates $R_{\varepsilon\varepsilon}(0)$ and $R_{X\varepsilon}(\mu)$ are determined from expressions (6), (9) and (10).

5. Algorithms for analysis of noisy vibration signals using equivalent samples of their noises and useful signals

The studies showed that it is possible to reduce the errors of the traditional methods of the correlation analysis of noise vibration signals using the technology for determining the equivalent samples of their noise $\varepsilon^e(i\Delta t)$ [1-3]. For this purpose, we first consider the possibility of calculating approximate quantities that cannot be directly measured by the6 samples of the noise $\varepsilon(i\Delta t)$ of the noise vibration signals $g(i\Delta t)$. An analysis of possible solutions to this problem demonstrated [1-3, 5-10] that, using the technology for calculating the estimate of the noise variance D_ε from expression (6), instead of immeasurable samples of the the noise $\varepsilon(i\Delta t)$, we can determine their approximate equivalent values $\varepsilon^e(i\Delta t)$. For this purpose, formula (6) is represented in the form:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right]$$

Due to this, taking the notation:

$$\varepsilon'(i\Delta t) = g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right]$$
$$\mathrm{sgn}\varepsilon'(i\Delta t) = \begin{cases} +1 & \text{when } \varepsilon'(i\Delta t) > 0 \\ 0 & \text{when } \varepsilon'(i\Delta t) = 0 \\ -1 & \text{when } \varepsilon'(i\Delta t) < 0 \end{cases},$$

the formula for calculating the equivalent values of samples of the noise $\varepsilon^e(i\Delta t)$ can be represented as follows:

$$\varepsilon(i\Delta t) \approx \varepsilon^e(i\Delta t) =$$
$$= \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{\left|g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right]\right|} =$$
$$= \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}. \quad (11)$$

Here, assuming that the following expression is true [1, 3]:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon^{e\,2}(i\Delta t) =$$
$$= \frac{1}{N}\sum_{i=1}^{N}\left|g(i\Delta t)\left[g(i\Delta t) + g\big((i+2)\Delta t\big) - 2g((i+1)\Delta t\right]\right|, \quad (12)$$

the formula for calculating the mean value $\bar{\varepsilon}(i\Delta t)$ of samples of the noise $\varepsilon(i\Delta t)$ can be reduced to the calculation of the mean value of the equivalent samples of the noise $\varepsilon^e(i\Delta t)$, т.е.:

$$\bar{\varepsilon}(i\Delta t) \approx \overline{\varepsilon^e}(i\Delta t) = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^e(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t).$$

Numerous experiments have shown that despite possible deviations of the approximate values of the equivalent samples $\varepsilon^e(i\Delta t)$ from their true values $\varepsilon(i\Delta t)$ by the value $\Delta\varepsilon(i\Delta t) = \varepsilon^e(i\Delta t) - \varepsilon(i\Delta t)$, the following equality takes place between their estimates:

$$P\left\{\frac{1}{N}\sum_{i=1}^{N}\varepsilon^{e^2}(i\Delta t) > \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t)\right\} \approx P\left\{\frac{1}{N}\sum_{i=1}^{N}\varepsilon^{e^2}(i\Delta t) < \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t)\right\} = 1,$$
$$P\left\{\frac{1}{N}\sum_{i=1}^{N}\varepsilon^{e}(i\Delta t) > \frac{1}{N}\sum_{i=1}^{N}\varepsilon(i\Delta t)\right\} \approx P\left\{\frac{1}{N}\sum_{i=1}^{N}\varepsilon^{e}(i\Delta t) < \frac{1}{N}\sum_{i=1}^{N}\varepsilon(i\Delta t)\right\} = 1. \quad (13)$$

Both equalities (11)-(13) and our experimental research demonstrate that by means of the equivalent samples of the noise $\varepsilon^e(i\Delta t)$, we can get results that are identical to the results of the analysis of the same vibration signals with known real samples of the noise $\varepsilon(i\Delta t)$. To this end, we use the formula:

$$X^e(i\Delta t) \approx g(i\Delta t) - \varepsilon^e(i\Delta t) \approx g(i\Delta t) - \varepsilon(i\Delta t) = X(i\Delta t)$$

to determine the equivalent samples $X^e(i\Delta t)$ of the useful vibration signal $X(i\Delta t)$.

In this case, the possibility also arises, by separating the equivalent noise samples $\varepsilon^e(i\Delta t)$ from the noisy vibration signal $g(i\Delta t)$ according to the obtained equivalent values of the samples of the useful signal $X^e(i\Delta t) = g(i\Delta t) - \varepsilon^e(i\Delta t)$, to determine the estimates $R^e{}_{XX}(\mu)$ and $R^e{}_{XX}(0)$ equivalent to the estimates of the correlation functions of the useful vibration signal $R_{XX}(\mu)$, i.e.:

$$R_{XX}(\mu) \approx \begin{cases} \frac{1}{N}\sum_{i=1}^{N} X^e(i\Delta t)X^e(i\Delta t) \; when\; \mu = 0 \\ \frac{1}{N}\sum_{i=1}^{N} X^e(i\Delta t)X^e((i+\mu)\Delta t) \; when\; \mu \neq 0 \end{cases}.$$

It is obvious that knowing the equivalent noise samples $\varepsilon^e(i\Delta t)$ and the useful signal $X^e(i\Delta t)$, we can determine the estimates of the cross-correlation function between the useful vibration signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ from the expression:

$$R_{X\varepsilon}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) \approx \frac{1}{N}\sum_{i=1}^{N} X^e(i\Delta t)\varepsilon^e\big((i+\mu)\Delta t\big)$$

Studies also showed that despite certain errors of the samples $X_i^e(i\Delta t)$ compared with the samples of useful signals $X(i\Delta t)$, with a sufficient duration of the observation time T, equality (13) holds. Due to this, the following equality is achieved:

$$R_{XX}(\mu) \approx R_{X^eX^e}(\mu),\; R_{X\varepsilon}(\mu) \approx R_{X^e\varepsilon^e}(\mu)$$

which shows that, using expressions (11) and (12), using equivalent samples of the noise $\varepsilon^e(i\Delta t)$ and of the useful signal $X^e(i\Delta t)$, we can find equivalent estimates of the correlation functions $R_{X^eX^e}(\mu)$

of the useful signal and the cross-correlation function $R_{X^e\varepsilon^e}(\mu)$ between the useful signal and the noise, which allow us to solve the problem of monitoring the beginning of changes in the technical condition of the track.

6. Vibration noise monitoring system

Fig. 1 shows the block diagram of a system of intelligent Noise monitoring, which consists of the following modules:

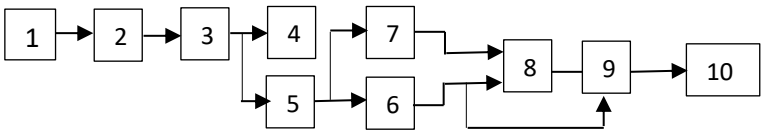

Fig. 1. Block diagram of the intelligent Noise monitoring system

1 – Vibration sensor; 2 – Module of sampling and formation of centered samples of the noise vibration signals $g(i\Delta t)$; 3 – Module of determining the estimates $R_{gg}(i\Delta t)$; 4 – Module of determining the equivalent samples of the useful signal $X(i\Delta t)$; 5 – Module of determining the estimates of the equivalent correlation functions $R_{X^eX^e}(\mu)$ and the cross-correlation function between the useful vibration signal and the noise $R_{X^e\varepsilon^e}(\mu)$ and the spectral estimates a_{nX^e}, b_{nX^e}, $a^{*e}_{n_\varepsilon}$, $b^{*e}_{n_\varepsilon}$ of the useful vibration signal $X(i\Delta t)$ and the noise $\varepsilon^e(i\Delta t)$; 6 – Module of formation of current informative attributes consisting of current estimates $R_{X^eX^e}(\mu)$ and $R_{X^e\varepsilon^e}(\mu)$, a_{nX^e}, b_{nX^e}, $a^{*e}_{n_\varepsilon}$, $b^{*e}_{n_\varepsilon}$, $R_{gg}(0)$; 7 – Learning module; 8 – Module of formation of the set of reference informative attributes; 9 – Decision-making module; 10 – Module of formation of information for signaling and remote transfer.

Fig. 1 shows a simplified block diagram of an intelligent vibration system for Noise monitoring of the technical condition of the railroad track. As can be seen from the block diagram in Fig. 1, the track Noise monitoring system is an inexpensive and fairly simple device, which practically consists of a vibration sensor, a sampling tool and a controller. Therefore, it can be installed in any cars of any train, and at the same time, does not require substantial costs.

The system operates as follows. At the beginning of the operation, the learning process begins and during the movement of the rolling stock in each cycle, by means of the appropriate modules, the sampled vibration signal $g(i\Delta t)$ is analyzed and the obtained estimates $R_{X^eX^e}(\mu)$, $R_{X^e\varepsilon^e}(\mu)$, D_{ε^e}, a_{nX^e}, b_{nX^e}, $a^*_{n_{\varepsilon^e}}$, $b^*_{n_{\varepsilon^e}}$ are saved as informative attributes. Subsequent estimates are compared with previous estimates and only maximum estimates are kept. The coefficients $K_1 = R_{X^\varepsilon X^\varepsilon}(\mu = 0)/R_{gg}(\mu = 0)$, $K_2 = D_\varepsilon/R_{X^\varepsilon X^\varepsilon}(\mu = 0)$, $K_3 = D_\varepsilon/R_{gg}(\mu = 0)$ are also taken as informative attributes. As a result, the set W_j^e forms after a certain time, which consists of maximum estimates of informative attributes that are taken as reference ones, i.e. $R^{max}_{X^eX^e}(\mu)$, $R^{max}_{X^e\varepsilon^e}(\mu)$, $D^{max}_{\varepsilon^e}$, $a^{max}_{nX^e}$, $b^{max}_{nX^e}$, $a^{*max}_{n_{\varepsilon^e}}$, $b^{*max}_{n_{\varepsilon^e}}$ K_1^{max}, K_2^{max}, K_3^{max}. In the following cycles, this process repeats and similarly forms the subsequent reference set.

If the current informative attributes are greater than the maximum reference attributes, then it is assumed that the training for a given haul is completed and the comparison of current combinations of informative attributes with an element of the set W_j of reference informative attributes begins. If the current attributes are not greater than the reference ones, then the technical condition of the track is considered unchanged. If current informative features are greater than the reference ones, it is assumed that the beginning of the latent period of changes in the technical condition of the track takes place. At the same time, information is formed in Module 10 to signal the advisability of control of the technical condition of a given haul using geometry cars. In the case when no change is detected,

it is also possible to form and transmit information about the safety of the track of this haul.

7. Conclusion

Modern geometry cars, flaw detector cars and other track test cars provide reliable control of the technical condition of all railroad hauls at certain intervals. Their number is limited and therefore "continuous control" of all hauls is practically impossible. Therefore, to ensure the safety of the track, they are used on schedule so that the control of each haul takes not less than a certain period of time. It is clear that the smaller this “gap”, the greater the guarantee of safety. However, in reality, it is impossible to guarantee the complete stability of the technical condition of the track during these periods of time. Therefore, it is impossible to guarantee complete safety of the track. Obviously, to solve this problem, it is necessary to monitor the beginning of changes in the technical condition of the track at the indicated time intervals using simple and inexpensive technical tools installed on the rolling stock, which will proceed to the corresponding hauls. This paper considers one of the possible solutions to this problem. An analysis of soil vibration caused by the impact of rolling stock forms informative attributes that can be used to determine the technical condition of the track. However, the use of traditional correlation and spectral analysis technologies proved to be ineffective for the formation of corresponding attributes of informative attributes by analyzing vibration signals. This is due to the fact that, substantial errors caused by the effects of the noise of vibration signals arise, decreasing the adequacy of the results of track control. In this paper, the technology of separate analysis of the useful vibration signal, the noise of the vibration signal, and the relationship between them is used to eliminate this difficulty, noise being used as a carrier of diagnostic information. To use the proposed Noise control, one of the possible options for technical monitoring tools is proposed, which can be easily implemented in all rolling stocks. This will make it possible to use them to monitor the beginning of changes in the technical condition of the track in all hauls during the movement of all trains.

References

[1] Путевые машины, применяемые ОАО «РЖД». Конструкция, теория и расчет. Глава 11. Машины и оборудование для диагностики пути. Региональный центр инновационных технологий. [In Russian: Track machines used by Russian Railways. Construction, theory and calculation. Chapter 11. Machines and equipment for track diagnostics. Regional Center for Innovative Technologies].

[2] Е.С. Ашпиз, Методы и средства диагностики земляного полотна, Железнодорожный путь, 2013. [In Russian: Y.S. Ashpiz, Methods and tools of roadbed diagnostics. Zheleznodorozhniy put].

[3] И.К. Михалкин, О.Б. Симаков, Ю.А. Седелкин, В.В. Атапин, Новые подходы к мониторингу железнодорожного пути, НПЦ ИНФОТРАНС. [In Russian: I.K. Mikhalkin, O.B. Simakov, Y.A. Sedelkin, V.V. Atapin, New approaches to monitoring the railroad track. NPTS INFOTRANS].

[4] Какие бывают вагоны-лаборатории и для чего они применяются? Available at: https://dvizhenie24.ru/railway/kakie-byvayut-laboratorii/ [In Russian: What kinds of track test cars are there and what are they used for?].

[5] Г.В. Осадчий, А.А. Лыков, Система диагностики и удаленного мониторинга состояния железнодорожного пути, Открытое образование. Изд. Федеральное государственное бюджетное образовательное учреждение высшего образования «Российский экономический университет имени Г.В. Плеханова». (2011) 221-224. [In Russian: Osadchiy, G.V. & Lykov, A.A. System for diagnostics and remote monitoring of the state of the railroad tracks. Open education. Federal State Budgetary Educational Institution of Higher Education “G.V. Plekhanov Russian University of Economics”].

[6] T. Aliyev, Noise Control of the Beginning and Development Dynamics of Accidents, Springer, 2019. 201 p. Available at: http://www.doi.org/10.1007/978-3-030-12512-7.

[7] T.A. Aliev, T. Babayev, T. Alizada, N. Rzayeva, Noise control of the beginning and development dynamics of faults in the running gear of the rolling stock, Transport Problems. 9 №10 (2019) 87-95.

[8] T.A. Aliev, H. Ahmedov, T. Babayev, T. Alizada, E. Manafov, N. Zohrabov, A. Mammadova, Using fuzzy set theory and noise analysis technologies to enhance validity and reliability of control of the condition of the running gear of rolling stock, In: Proceedings of the V Georgian-Polish International Scientific-Technical Conference

"Transport Bridge Europe-Asia" Kutaisi, 15-18 October 2019.

[9] T.A. Aliev, T. Babayev, T. Alizada, N. Rzayeva, Possibilities of application of noise technology in railroad operation safety systems in seismically active regions, 26-28 June 2019. The Silesian University of Technology, In: Transport Problems 2019: Conference Proceedings: XI International Scientific Conference: 26-28.06.2019, Katowice, Bochnia.