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Generalized numerical method for computer simulation of transient processes in complex electrical circuits with distributed parameters with an allowance for losses

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A B S T R A C T

On the basis of the development of the theory of operational calculus, a new generalized numerical method is presented for computer simulation of transient processes in complex electrical circuits with distributed parameters with an allowance for losses, when replacing the operation of continuous integration by summing, using the Simpson formula. New recurrence relations are obtained that are easily implemented on a computer.

1. Introduction

As our analysis shows, the operation mode of complex electrical circuits with distributed parameters during operation is mainly variable [1-6, 9-16, 18, 19].

At the same time, the isolation level of this system is largely determined by internal overvoltages arising under various operating conditions.

In this regard, the issues of studying transient processes in complex electrical circuits with distributed parameters that arise in various normal pre-emergency and emergency situations are of great practical interest for modeling transient processes in complex electrical circuits with distributed parameters, attract increasing attention.

The use of computer technology for numerical simulation of transient processes in complex electrical circuits with distributed parameters allows you to more accurately determine the shape of the distortion of the signals and parameters of the measuring device to receive signals in the desired form.

A new approach based on further development and generalization for computer simulation of transient processes in objects with distributed parameters described in partial derivatives of a hyperbolic type is the new numerical method proposed in [14-19], the essence of which is based on the use of a discrete analogue of the integral convolution equation.

The advantage of this numerical method is that it allows computer simulation of transient processes in systems with distributed parameters, without moving to the domain of discrete images [7, 9-13], and also transfers from the Laplace transform of the desired functions to the domain of originals without finding roots of characteristic equations, without expanding the operator wave

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propagation coefficient and operator wave impedance in series, which greatly simplifies the mathematical calculations and increases calculation accuracy.

In addition, the proposed new approach [14-19], in contrast to the existing methods [1-6, 8, 9-13], depending on the specified calculation accuracy, allows replacing the operation of continuous integration by summing, using not only rectangle formulas, but also trapezoidal, Simpson, Weddle.

These properties of the new approach [14-19] significantly expand the range of problems.

This paper a further generalization and development of works [14-19] for computer simulation of transient processes in complex electrical circuits with distributed parameters with an allowance for losses, when replacing the operation of continuous integration by summation, in the general case, using the Simpson formula.

2. Problem statement

Let us consider the process of switching on a loaded complex electric circuit with distributed parameters with a lumped inductance L_2, active resistance R_2 at the end, to an arbitrary voltage source U_o (t) through the lumped resistance R_1 and inductance L_1 (Fig.1).

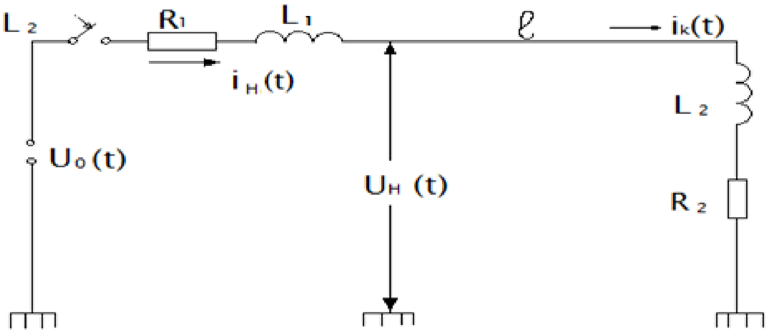

Fig. 1

Transient processes occurring in electrical circuits with distributed parameters are generally described by telegraph equations [1-4]:

$$\frac{-\partial u}{\partial t} = L\frac{\partial i}{\partial t} + Ri$$
$$-\frac{\partial i}{\partial x} = C\frac{\partial u}{\partial t} + Gu, \qquad 0 \le x \le l \tag{1}$$

where $u = u(x,t)$ is voltage; $i = i(x,t)$ is current; L, C, R, G is resistance, inductance, conductivity and capacitance between wire and ground, referred to a unit of circuit length.

The initial conditions are zero:

$$u(x,t)_{t=0} = 0, i(x,t)_{t=0} = 0$$

The boundary conditions have the form:

$$u(x,t)_{x=0} = u_H(t), \quad u_k(t) = R_2 i_k(t) + L_2\frac{di_k(t)}{dt},$$

where $u_k(t) = u(l,t), \ \ i_k(t) = i(l,t),$

3. Solution

In solving the problem at the first stage, it is necessary to obtain a Laplace transform for the functions $i(x,t), u(x,t)$.

Using this method, under the adopted initial and boundary conditions, from the solution of the system of differential equations (1) we obtain the following expressions for the indicated functions in operator form:

$$I(x,s)=\frac{1}{\rho(s)}\frac{sh\gamma(l-x)}{ch\gamma l}U_H(s)+I_k(s)\frac{ch\gamma x}{sh\gamma l} \tag{2}$$

$$U(x,s)=\frac{ch\gamma(l-x)}{ch\gamma l}U_H(s)-\rho(s)I_k(s)\frac{sh\gamma x}{ch\gamma l}, \tag{3}$$

where $\gamma=\gamma(s)=\sqrt{(sL+R)(sC+G)}$ is the operator constant of the propagation of the wave $\rho(s)=\sqrt{\frac{sL+R}{sC+G}}$ is the operator wave impedance of the circuit; s is the Laplace transform operator; $\mathrm{U(x,s)},\mathrm{I(x,s)},U_H(\mathrm{s}),I_{k(s)}$ is a Laplace transform of the functions $\mathrm{u(x,t)},\mathrm{i(x,t)},u_H(t),i_k(t)$.

The second stage of solving this problem is related to the transition from Laplace transform (2), (3) to the domain of originals.

In this regard, in the expressions for the functions $I(x,s),U(x,s)$ from (2), (3), proceeding from hyperbolic functions to power functions, we obtain:

$$I(\delta,s)\frac{1}{s}=\frac{1}{\rho}\left(+\frac{G}{C}\frac{1}{s}\right)\frac{1}{\sqrt{(s+\alpha)^2-\beta^2}}\frac{e^{-2\gamma l\delta}-e^{-2\gamma l(1-\delta)}}{1+e^{-2\gamma l}}U_H(s)+$$
$$+\frac{1}{s}\frac{e^{-\gamma l(1-2\delta)}+e^{-\gamma l(1+2\delta)}}{1+e^{-2\gamma l}}I_k(s) \tag{4}$$

$$U(\delta,s)\left(1+\frac{G}{C}\frac{1}{s}\right)\frac{1}{\sqrt{(s+\alpha)^2-\beta^2}}=\left(1+\frac{G}{C}\frac{1}{s}\right)\frac{1}{\sqrt{(s+\alpha)^2-\beta^2}}\frac{e^{-2\gamma l\delta}+e^{-2\gamma l(1-\delta)}}{1+e^{-2\gamma l}}U_H(s)-$$
$$-\rho I_k(s)\frac{e^{-\gamma l(1-2\delta)}-e^{-\gamma l(1+2\delta)}}{1+e^{-2\gamma l}}\frac{1}{s}\ , \tag{5}$$

where $\gamma=\gamma(s)=\frac{1}{v}\sqrt{(s+\alpha)^2-\beta^2}\,,v=1/\sqrt{LC}$ is the wave propagation speed; $\rho=\sqrt{\frac{L}{C}}$ is the wave impedance of a radio circuit with distributed parameters without loss, $\alpha=\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)$ is the attenuation coefficient; $\beta=\frac{1}{2}\left(\frac{R}{L}-\frac{G}{C}\right)$ is the distortion factor; $\delta=\frac{x}{2l}$.

In the particular case, if $\mathrm{G}=0$, then $\alpha=\beta$. At $\beta=0$, in the so-called balanced radio circuits with distributed parameters, the following ratio of parameters takes place $\frac{R}{L}=\frac{G}{C}$.

For the balanced case $\beta=0$, the coefficient r is equal to the same value as for the case without loss in the line.

Expressions (4), (5) can be represented as

$$I(\delta,s)\left[\frac{1}{s}+k_1(s)\right]=\frac{1}{\rho}\left[k_2(s)+\frac{G}{C}k_3(s)-k_4(s)-\frac{G}{C}k_5(s)\right]U_H(s)+$$
$$+[\mathrm{k}_6(\mathrm{s})+\mathrm{k}_7(\mathrm{s})]\mathrm{I}_\mathrm{k}(\mathrm{s}) \tag{6}$$

$$U(\delta,s)\left[k_8(s)+\frac{G}{C}k_9(s)+k_{10}(s)+\frac{G}{C}k_{11}(s)\right]=$$
$$=\left[k_2(s)+\frac{G}{C}k_3(s)+k_4(s)+\frac{G}{C}k_5(s)\right]U_H(s)-\rho[k_6(s)-k_7(s)]I_k(s), \tag{7}$$

where $k_1(s)=\frac{e^{-2\gamma l}}{s},k_2(s)=\frac{e^{-2\gamma l}}{\sqrt{(s+\alpha)^2-\beta^2}},k_3(s)=\frac{1}{s}\cdot\frac{e^{-2\gamma l\delta}}{\sqrt{(s+\alpha)^2-\beta^2}},$

$$k_4(s)=\frac{e^{-2\gamma l(1-\delta)}}{\sqrt{(s+\alpha)^2-\beta^2}},\qquad k_5(s)=\frac{1}{s}\cdot\frac{e^{-2\gamma l(1-\delta)}}{\sqrt{(s+\alpha)^2-\beta^2}},$$

$$k_6(s)=\frac{e^{-\gamma l(1-2\delta)}}{s},\qquad k_7(s)=\frac{e^{-\gamma l(1-2\delta)}}{s},k_8(s)=\frac{1}{\sqrt{(s+\alpha)^2-\beta^2}},$$

$$k_9(s)=\frac{1}{s}\frac{1}{\sqrt{(s+\alpha)^2-\beta^2}},k_{10}(s)=\cdot\frac{e^{-2\gamma l}}{\sqrt{(s+\alpha)^2-\beta^2}}$$

$$k_{11}(s)=\frac{1}{s}\frac{e^{-2\gamma l}}{\sqrt{(s+\alpha)^2-\beta^2}}$$

$k_1(s),\dots,k_{11}(s)$ is transfer functions.

Based on the convolution theorem, proceeding from equations (6), (7) relative to images to equations relative to the originals, we obtain:

$$\int_0^t i(t-\theta,\delta)1(\theta)d\theta+\int_{\frac{2l}{v}}^t i(t-\theta,\delta)k_1(\theta)d\theta=\frac{1}{\rho}\left[\int_{\frac{2l\delta}{v}}^t u_H(t-\theta)k_2(\theta)d\theta+\right.$$
$$+\frac{G}{C}\int_{\frac{2l\delta}{v}}^t u_H(t-\theta)\,k_3(\theta)d\theta-\int_{\frac{2l(1-\delta)}{v}}^t u_H(t-\theta)k_4(\theta)d\theta-\frac{G}{C}\int_{\frac{2l(1-\delta)}{v}}^t u_H(t-\theta)k_3(\theta)d\theta]+$$
$$+\int_{\frac{l(1-2\delta)}{v}}^t i_k\,(t-\quad-\theta)k_6(\theta)d\theta\int_{\frac{l(1+2\delta)}{v}}^t i_k(t-\theta)k_7\,(\theta)d\theta \tag{8}$$

$$\int_0^t i(t-\theta,\delta)k_8(\theta)d\theta+\frac{G}{C}\int_0^t u(t-\theta,\delta)k_9\,(\theta)d\theta+\int_{\frac{2l}{v}}^t u(t-\theta,\delta)k_{10}(\theta)d\theta+$$
$$+\frac{G}{C}\int_{\frac{2l}{v}}^t \omega(t-\theta,\delta)\,k_{11}(\theta)d\theta=\int_{\frac{2l\delta}{v}}^t u_H(t-\theta)k_2(\theta)d\theta+\frac{G}{C}\int_{\frac{2l\delta}{v}}^t u_H(t-\theta)k_3\,(\theta)d\theta+$$
$$+\int_{\frac{2l(1-\delta)}{v}}^t u_H(t-\theta)\,k_4(\theta)d\theta+\frac{G}{C}\int_{\frac{2l(1-\delta)}{v}}^t u_H(t-\theta)k_5(\theta)d\theta-\rho\left(\int_{\frac{l(1-2\delta)}{v}}^t i_k(t-\theta)\,k_6(\theta)d\theta-\right.$$
$$-\int_{\frac{l(1+2\delta)}{v}}^t i_k(t-\theta)\,k_7(\theta)d\theta), \tag{9}$$

where $k_1(t),\dots,k_{11}(t)$ are known originals of the transfer functions. $k_1(s),\dots,k_{11}(s)$.

Integral equations (8), (9) can be solved numerically if the integrals are replaced by the sums.

In this regard, using the relationship between continuous time t and discrete n $(n=0,1,2\dots)$ in the form $t=nT/\lambda$ [9] (where λ is any integer, $T=2\tau,\tau=\frac{l}{v}$ is the propagation time of the wave at one end of the chain with distributed parameters), we sample equations (8), (9) for the selected interval T/λ, replacing the operation of continuous integration by summing using the Simpson formula.

Here, instead of (8),(9) with respect to the lattice functions $\mathrm{i}[\mathrm{n},\delta],\mathrm{u}[\mathrm{n},\delta]$, we obtain the following recurrence relations:

$$i[n,\delta]=\frac{1}{\rho}\Bigg[\sum_{m=\lambda\delta}^{n}((k_2[m]+\frac{G}{C}\frac{T}{3\lambda}k_3[m])U_H[n-m]\ +4(k_2[m-1]+$$

$$+\frac{G}{C}\frac{T}{3\lambda}k_3[m-1])U_H[n-m+1]+\left(k_2[m-2]+\frac{G}{C}\frac{T}{3\lambda}k_3[m-2]\right)U_H[n-m+2])-$$

$$-\sum_{m=\lambda(1-\delta)}^{n}((\left(k_4[m]+\frac{G}{C}\frac{T}{3\lambda}k_5[m]\right)U_H[n-m]+4(k_4[m-1]+$$

$$+\frac{G}{C}\frac{T}{3\lambda}k_5[m-1])U_H[n-m+1]+\left(k_4[m-2]+\frac{G}{C}\frac{T}{3\lambda}k_5[m-2]\right)U_H[n-m+2])\Bigg]+$$

$$+\sum_{m=0,5\lambda(1-2\delta)}^{n}(\,k_6[m]\,i_k[n-m]+4k_6[m-1]\,i_k[n-m+1]+k_6[m-2]i_k[n-m+2])+$$

$$+\sum_{m=0,5\lambda(1+2\delta)}^{n}(k_7[m]i_k[n--m]+4k_7[m-1]i_k[n-m+1]+k_7[m-2]i_k[n-m+2])-$$

$$-\sum_{m=\lambda}^{n}(k_1[m]i[n-m,\delta]++4k_1[m-1]i[n-m+1,\delta]+k_1[m-2]i[n-m+2,\delta])-$$

$$-\sum_{m=1}^{n}(1[m]i[n-m,\delta]+4\cdot 1[n-m+1]i[m-1,\delta]+1[n-m+2]i[m-2,\delta]) \qquad (10)$$

$$\mathrm{U}[\mathrm{n},\delta]=A_0\left\{\sum_{\mathrm{m}=\lambda\delta}^{\mathrm{n}}((\mathrm{k}_2[\mathrm{m}]+\frac{\mathrm{G}}{\mathrm{C}}\cdot\frac{\mathrm{T}}{3\lambda}\mathrm{k}_3[\mathrm{m}]\right)\mathrm{U_H}[\mathrm{n}-\mathrm{m}]+4(\mathrm{k}_2[\mathrm{m}-1]+$$

$$+\frac{\mathrm{G}}{\mathrm{C}}\cdot\frac{\mathrm{T}}{3\lambda}\mathrm{k}_3[\mathrm{m}-1])\mathrm{U_H}[\mathrm{n}-\mathrm{m}+1]+(\mathrm{k}_2[\mathrm{m}-2]+\frac{\mathrm{G}}{\mathrm{C}}\cdot\frac{\mathrm{T}}{3\lambda}\mathrm{k}_3[\mathrm{m}-2]\mathrm{U_H}[\mathrm{n}-\mathrm{m}+2])+$$

$$+\sum_{m=\lambda(1-\delta)}^{n}((\,k_4[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5[m]U_H[n-m])+4(k_4[m-1]+$$

$$+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5[m-1])U_H[n-m+1]+\left(k_4[m-2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5[m-2]\right)U_H[n-m+2])-$$

$$-\rho\Bigg[\sum_{m=0,5\lambda(1-2\delta)}^{n}(k_6\,[m]i_k[n-m]+4k_6[m-1]i_k[n-m+1]+k_6[m-2]i_k[n-m+2])-$$

$$-\sum_{m=0,5\lambda(1+2\delta)}^{n}(k_7[m]i_k\,[n-m]+4k_7[m-1]i_k[n-m+1]+k_7[m-2]i_k[n-m+2])]-$$

$$-\sum_{m=\lambda}^{n}\left(\left(k_{10}[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_{11}[m]\right)U[n-m,\delta]+4(k_{10}\,[m-1]+\right.$$

$$+\frac{G}{C}\cdot\frac{T}{3\lambda}k_{11}[m-1])U[n-m+1,\delta]+\left(k_{10}[m-2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_{11}[m-2]\right)U[n-m+2,\delta])\text{-}$$

$$-\sum_{m=1}^{n}\left(k_8[(m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[m])U[n-m,\delta]+\right.$$

$$+4\left(k_8[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+1]\right)U[m-1,\delta]+$$

$$+\left(k_8[n-m+2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+2]\right)U[m-2,\delta]), \qquad (11)$$

where $\frac{G}{C}\frac{T}{3\lambda}=\frac{1}{3\lambda}(\alpha T+\beta T)$, $A_0=\frac{1}{1+\frac{GT}{C\lambda}}$,

$$k_1[\mathrm{n}]=\begin{cases}0, & \text{when } n<\lambda\\ e^{-\alpha T}+\frac{\alpha T}{3}\sum_{m=\lambda+1}^{n}(\beta_1[m]+4\beta_1[m-1]+\beta_1[m-2]), & \text{when } n>\lambda\end{cases}$$

$$\beta_1[n]=e^{-\frac{\alpha T}{\lambda}n}\frac{I_1\left(\frac{\beta T}{\lambda}\sqrt{n^2-\lambda^2}\right)}{\sqrt{n^2-\lambda^2}}, k_2[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-(\lambda\delta)^2}\right),$$

$$k_3[n]=\sum_{m=\lambda\delta}^{n}(k_2[m]+4k_2[m-1]+k_2[m-2]),$$

$$k_4[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-[\lambda(1-\delta)]^2}\right),$$

$$k_5[n]=\sum_{m=\lambda(1-\delta)}^{n}(k_4[m]+4k_4[m-1]+k_4[m-2]),$$

$$k_6[n]=\begin{cases}0, & \text{when } n<0{,}5\lambda(1-2\delta)\\ e^{-\alpha\tau(1-2\delta)}+\frac{2\tau(1-2\delta)}{3}\sum_{m=0{,}5\lambda(1-2\delta)+1}^{n}(\beta_6[m]+4\beta_6[m-1]+\beta_6[m-2]), & \text{when } 0{,}5\lambda(1-2\delta)\end{cases}$$

$$\beta_6[n]=e^{\frac{-\alpha T}{\lambda}n}\frac{I_1\left(\frac{\beta T}{\lambda}\sqrt{n^2-[0{,}5\lambda(1-2\delta)^2)]}\right)}{\sqrt{n^2-(0{,}5\lambda(1+2\delta))^2}}$$

$$k_7[n]=\begin{cases}0, & \text{when } n<0{,}5\lambda(1+2\delta)\\ e^{-\alpha T(1+2\delta)}+\frac{\alpha\tau(1+2\delta)}{3}\sum_{m=0.5\lambda(1+2\delta)+1}^{n}(\beta_7[m]+4\beta_7[m-1]+\beta_7[m-2]), & \text{when } n>0{,}5\lambda(1+2\delta)\end{cases}$$

$$\beta_7=e^{\frac{-\alpha T}{\lambda}n}\frac{I_1\left(\frac{\beta T}{\lambda}\sqrt{n^2-(0{,}5\lambda(1+2\delta))^2}\right)}{\sqrt{n^2-(0{,}5\lambda(1+2\delta))^2}}$$

$$k_8[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\frac{\beta T}{\lambda}n\right),\quad k_9[n]=\sum_{m=0}^{n}(k_8[m]+4k_8[m-1]+k_8[m-2]),$$

$$k_{10}[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\frac{\beta T}{\lambda}\sqrt{n^2-\lambda^2}\right), k_{11}[n]=\sum_{m=\lambda}^{n}(k_{10}[m]+4k_{10}[m-1]+k_{10}[m-2])$$

Obtained recurrence relations (10), (11) are easily implemented on a computer.

The calculation error is related to the quantity λ. The larger the number λ is selected, the less the characteristics of continuous functions differ from the corresponding characteristics of lattice functions.

The recurrence relations (10), (11) include unknown functions $U_H[\mathrm{n}], i_k[\mathrm{n}]$.. Their values is determined according to the following procedure.

For the starting point of an electric circuit with distributed parameters, the following expression can be represented in operator form:

$$i_H(\mathrm{S})=[U_0(\mathrm{s})-U_H(\mathrm{s})]k_0(\mathrm{s}), \qquad (12)$$

where $K_0(\mathrm{s}) = \frac{1}{R_1 + L_1 S}$

Expression (12) based on the convolution theorem in the domain of originals can be represented as:

$$i_H(t) = \frac{1}{L_1}\int_0^t k_0(\theta_1)[U_0(t-\theta_1) - U_H(t-\theta_1)]\,\mathrm{d}\theta_1, \tag{13}$$

where $k_0(t) = e^{-\frac{R_1}{L_1}t}$

Expression (13) based on the Simpson formula in lattice form will be:

$$i_n[n] = \frac{T}{3\lambda L_1}\sum_{m=0}^{n}(k_0[m](U_0[n-m] - U_H[n-m]) +$$
$$+4k_0[n-m+1](U_0[m-1] - U_H[m-1]) +$$
$$+k_0[n-m+2](U_0[m-2] - U_H[m-2])), \tag{14}$$

where $k_0[n] = e^{-\frac{TR_1}{\lambda L_1}n}$

Here,

$$\sum_{m=0}^{n}(k_0[m](U_0[n-m] - U_H[n-m]) + 4k_0[n-m+1](U_0[m-1] - U_H[m-1]) +$$
$$+k_0[n-m+2](U_0[m-2] - U_H[m-2])) = \mathrm{U}_0[\mathrm{n}] - \mathrm{U}_{\mathrm{H}}[n] +$$
$$+\sum_{m=1}^{n}(\,k_0[m](\mathrm{U}_0[n-m] - \,\mathrm{U}_H\,[n-m]) + 4K_0[n-m+1](U_0[m-1] - \,U_H[m-1]) +$$
$$+k_0[n-m+2](U_0[m-2] - \,U_H[m-2])) = U_0[n] - U_H[n] + B[n], \tag{15}$$

where

$$B[n] = \sum_{m=1}^{n}(k_0[m](U_0[n-m] - U_H[n-m]) + +4k_0[n-m+1](U_0[m-1] -$$
$$-U_H[m-1]) + k_0[n-m+2](U_0[m-2] - U_H[m-2]))$$

Expression (14) with allowance for (15) can be represented as:

$$i_H[n] = \frac{T}{3\lambda L_1}(U_0[n] - U_H[n] + B[n]) \tag{16}$$

For $\delta = 0$, from recurrence relation (10), we can present the following expression for the current $i_{\mathrm{H}}[n]$:

$$i_H[n] = \frac{1}{\rho}\sum_{m=o}^{n}\left(\left(k_2'[m] + \frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[m]\right)U_H[n-m] +\right.$$
$$+4\left(k_2'[n-m+1] + \frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+1]\right)U_H[m-1] +$$
$$+\left(U_H[n-m+2] + \frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+2]\right)U_H[m-2]) -$$

$$-\sum_{m=\lambda}^{n}\Big((k_4'[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5'[m]U_H[n-m]+4(k_4[m-1]+$$
$$+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5'[m-1])U_H[n-m+1]+\Big(k_4'[m-2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_5'[m-2]\Big)U_H[n-m+2])]+$$
$$+2\sum_{m=0,5\lambda}^{n}(k_6'[m]i_k[n-m]+4k_6'[m-1]i_k[n-m+1]++k_6'[m-2]i_k[n-m+2])-$$
$$-\sum_{m=\lambda}^{n}(k_1[m]i_H[n-m]+4k_1[m-1]i_H[n-m+1]+k_1[m-2]i_H[n-m+2])-$$
$$-\sum_{m=1}^{n}(1[m]i_H[n-m]+4\cdot 1[n-m+1]i_H[m-1]+1[n-m+2]i_H[m-2]), \qquad (17)$$

where

$$k_2'[n]=e^{-\frac{\alpha T}{\lambda}n}I_0(\beta\frac{T}{\lambda}n),$$
$$k_3'[n]=\sum_{m=0}^{n}\big(k_2'[m]+4k_2'[m-1]+k_2'[m-2]\big),$$
$$k_4'[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-\lambda^2}\right),$$
$$k_5'[n]=\sum_{m=\lambda}^{n}(k_4'[m]+4k_4'[m-1]+k_4'[m-2])$$
$$k_6'[n]=\begin{cases}0, & \text{when } n<0,5\lambda\\ e^{-\alpha\tau}+\frac{\alpha\tau}{3}\sum\limits_{m=0,5\lambda+1}^{n}(\beta_6'[m]+4\beta_6'[m-1]+\beta_6'[m-2]), & \text{when } n>0,5\lambda\end{cases}$$
$$\beta_6'[n]=e^{-\frac{\alpha T}{\lambda}n}\frac{I_1\left(\frac{\beta T}{\lambda}\sqrt{n^2-(0,5\lambda)^2}\right)}{\sqrt{n^2-(0,5\lambda)^2}}$$

Here,

$$\sum_{m=0}^{n}\left(\left(k_2'[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[m]\right)U_H[n-m]+\right.$$
$$+4\left(k_2'[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+1]\right)U_H[m-1]+(k_2'[n-m+2]+$$
$$+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+2])U_H[m-2])=\left(k_2'[0]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[0]\right)U_H[n]+$$
$$+\sum_{m=1}^{n}\big((k_2'[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+1])U_H[m-1]+$$
$$+4\left(k_2'[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+1]\right)U_H[m-1]+$$
$$+\left(k_2'[n-m+2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_3'[n-m+2]\right)U_H[m-2]), \qquad (18)$$

where $k_2'[0]=k_3'[0]=1$

Expression (17) with allowance for (18) can be represented as:

$$i_H[n] = \frac{1}{\rho}\left(1 + \frac{G}{C} \cdot \frac{T}{3\lambda}\right) U_H[n] + B_1[n], \tag{19}$$

where

$$B_1[n] = \frac{1}{\rho}\Bigg[\sum_{m=1}^{n} \left(\left(k_2'[m] + \frac{G}{C} \cdot \frac{T}{3\lambda} k_3'[m]\right) U_H[n-m] + \right.$$
$$+4\left(k_2'[n-m+1] + \frac{G}{C} \cdot \frac{T}{3\lambda} k_3'[n-m+1]\right) U_H[m-1] + (k_2'[n-m+2] +$$
$$\left. + \frac{G}{C} \cdot \frac{T}{3\lambda} k_3'[n-m+2]\right) U_H[m-2]) -$$
$$-\sum_{m=\lambda}^{n} ((k_4'[m] + \frac{G}{C} \cdot \frac{T}{3\lambda} k_5'[m]) U_H[n-m] + 4[k_4'[m-1] +$$
$$+ \frac{G}{C} \cdot \frac{T}{3\lambda} k_5'[m-1]) \, U_H[n-m+1] + (k_4'[m-2] + \frac{G}{C} \cdot \frac{T}{3\lambda} k_5'[m-2]) \, U_H[n-m+2]) +$$
$$+2 \sum_{m=0,5\lambda}^{n} (k_6'[m] i_k[n-m] + 4\, k_6'[m-1] i_k[n-m+1] + k_6'[m-2] i_k[n-m+2]) -$$
$$-\sum_{m=\lambda}^{n} (k_1[m] i_H[n-m] + 4k_1[m-1] i_H[n-m+1] + k_1[m-2] i_H[n-m+2]) -$$
$$-\sum_{m=1}^{n} (1[m] i_H[n-m] + 4 \cdot 1[n-m+1] i_H[m-1] + 1[n-m+2]\, i_H[m-2])$$

Substituting the value of the function $i_H[n]$ from (19) into (16), we obtain the following expression for the voltage $U_H[n]$:

$$U_H[n] = \frac{1}{A + A_1} \{A(U_0[n] + B[n]) - B_1[n]\}\,, \tag{20}$$

where

$$A = \frac{T}{3\lambda L_1}, A_1 = \frac{1}{\rho}\left(1 + \frac{G}{C} \cdot \frac{T}{3\lambda}\right)$$

In recurrence relation (20), the expression for the lattice function $B_1[n]$ includes the unknown function $i_k[n]$ is a change in current at the end of an electric circuit with distributed parameters. Its value is determined as follows.

According to the boundary conditions, we can represent the following expression for the function $U_k(t)$ in operator form:

$$U_k(s) = (R_2 + L_2 S) I_k(s) \tag{21}$$

Expression (21) can be represented as:

$$I_k(s) = W_1(s) U_k(s), \tag{22}$$

where $W_1(s) = \frac{1}{L_2 s + R_2}$

Expression (22) based on the convolution theorem in the domain of originals can be represented as:

$$i_k[n] = \frac{1}{L_2} \int_0^t W_1\,(\theta_1) U_k(t - \theta_1) d\theta_1, \tag{23}$$

where $W_1(t) = e^{-\frac{R_2}{L_2}t}$

Expression (23) based on the Simpson formula in lattice form will be:

$$i_k[n] = \frac{T}{3\lambda L_2}\sum_{m=0}^{n}(W_1[m]U_k[n-m] + 4W_1[n-m+1]U_k[m-1] + $$
$$+W_1[n-m+2]U_k[m-2]), \qquad (24)$$

where $W_1[n] = e^{-\frac{TR_2}{\lambda L_2}n}$

Here,

$$\sum_{m=0}^{n}(W_1[m]U_k[n-m] + 4W_1[n-m+1]U_k[m-1] + W_1[n-m+2]U_k[m-2]) =$$
$$= W_1[0]U_k[n] + \sum_{m=1}^{n}(W_1[m]U_k[n-m] + 4W_1[n-m+1]U_k[m-1] +$$
$$+W_1[n-m+2]U_k[m-2]) \qquad (25)$$

Expression (24) taking into account (25) will be

$$i_k[n] = \frac{T}{3\lambda L_2}\Bigg(U_k[n] + \sum_{m=1}^{n}(W_1[m]U_k[n-m] + 4W_1[n-m+1]U_k[m-1] +$$
$$+W_1[n-m+2]U_k[m-2])\Bigg) \qquad (26)$$

In expression (26), the value of voltage $U_k[n]$ is determined from the recurrence relation (11) for $\delta = \frac{1}{2}$, then:

$$U_k[n] = A_0\Bigg\{2\sum_{m=0,5\lambda}^{n}\left(\left(k_2'[m] + \frac{G}{C}\frac{T}{3\lambda}k_3'[m]\right)U_H[n-m] +\right.$$
$$+4\left(k_2'[m-1] + \frac{G}{C}\frac{T}{3\lambda}k_3'[m-1]\right)U_H[n-m+1] +$$
$$+\left(k_2'[m-2] + \frac{G}{C}\frac{T}{3\lambda}k_3'[m-2]\right)U_H[n-m+2]) -$$
$$-\rho\left[\sum_{m=0}^{n}(k_6'[m]i_k[n-m] + 4k_6'[n-m+1]i_k[m-1] + k_6'[n-m+2]i_k[m-2]) -\right.$$
$$\left.-\sum_{m=\lambda}^{n}\left(k_7'[m]i_k[n-m] + 4k_7'[m-1]i_k[n-m+1] + k_7'[m-2]i_k[n-m+2]\right)\right] -$$
$$-\sum_{m=\lambda}^{n}\left(\left(k_{10}[m] + \frac{G}{C}\frac{T}{3\lambda}k_{11}[m]\right)U_k[n-m] +\right.$$
$$+4\left(k_{10}[m-1] + \frac{G}{C}\frac{T}{3\lambda}k_{11}[m-1]\right)U_k[n-m+1] +$$
$$+\left(k_{10}[m-2] + \frac{G}{C}\frac{T}{3\lambda}k_{11}[m-2]\right)U_k[n-m+2]) -$$

$$-\sum_{m=1}^{n}\Bigg(\left(k_8[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[m]\right)U_k[n-m]+$$
$$+4\left(k_8[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+1]\right)U_k[m-1]+$$
$$+\left(k_8[n-m+2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+2]\right)U_k[m-2]\Bigg)\Bigg\}, \qquad (27)$$

where $k_2'[n]=e^{-\frac{\alpha T}{\lambda}n}I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-(0{,}5\lambda)^2}\right)$,

$$k_3'[n]=\sum_{m=0.5\lambda}^{n}\left(k_2'[m]+4k_2'[m-1]+k_2'[m-2]\right), \qquad k_6'[n]=1$$

$$k_7'[n]=\begin{cases}0, & \text{when } n<\lambda\\ e^{-2\alpha\tau}+\dfrac{2\alpha\tau}{3}\displaystyle\sum_{m=\lambda+1}^{n}\left(\beta_7'[m]+4\beta_7'[m-1]+\beta_7'[m-2]\right), & \text{when } n>\lambda\end{cases}$$

$$\beta_7'[n]=e^{-\frac{\alpha T}{\lambda}n}\frac{I_1\left(\frac{\beta T}{\lambda}\sqrt{n^2-\lambda^2}\right)}{\sqrt{n^2-\lambda^2}}$$

Recurrence relation (27) can be represented as:

$$U_k[n]=B_1'[n]-\rho A_0\sum_{m=0}^{n}\left(1[m]\,i_k[n-m]+4\cdot 1[n-m+1]i_k[m-1]+\right.$$
$$\left.+1[n-m+2]i_k[m-2]\right), \qquad (28)$$

where

$$B_1'[n]=A_0\Bigg\{2\sum_{m=0,5\lambda}^{n}\Bigg(\left(k_2'[m]+\frac{G}{C}\frac{T}{3\lambda}k_3'[m]\right)u_H[n-m]+$$
$$+4\left(k_2'[m-1]+\frac{G}{C}\frac{T}{3\lambda}k_3'[m-1]\right)u_H[n-m+1]+$$
$$+\left(k_2'[m-2]+\frac{G}{C}\frac{T}{3\lambda}k_3'[m-2]\right)u_H[n-m+2]\Bigg)-$$
$$-\sum_{m=\lambda}^{n}\left(k_7'[m]i_k[n-m]+4k_7'[m-1]i_k[n-m+1]+k_7'[m-2]i_k[n-m+2]\right)-$$
$$-\sum_{m=\lambda}^{n}\Bigg(\left(k_{10}[m]+\frac{G}{C}\frac{T}{3\lambda}k_{11}[m]\right)U_k[n-m]+$$
$$+4\left(k_{10}[m-1]+\frac{G}{C}\frac{T}{3\lambda}k_{11}[m-1]\right)U_k[n-m+1]+$$
$$+\left(k_{10}[m-2]+\frac{G}{C}\frac{T}{3\lambda}k_{11}[m-2]\right)U_k[n-m+2]\Bigg)-$$
$$-\sum_{m=1}^{n}\Bigg(\left(k_8[m]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[m]\right)u_k[n-m]+$$
$$+\left(k_8[n-m+1]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+1]\right)u_k[m-1]+$$
$$+\left(k_8[n-m+2]+\frac{G}{C}\cdot\frac{T}{3\lambda}k_9[n-m+2]\right)u_k[m-2]\Bigg)\Bigg\}$$

Expression (28) can be represented as:

$$U_k[n] = B_2[n] - \rho A_0 i_k[n]\,, \tag{29}$$

where

$$B_2[n] = B_1'[n] - \rho A_0 \sum_{m=1}^{n} (1[m] i_k[n-m] + 4 \cdot 1[n-m+1] i_k[m-1] +$$
$$+1[n-m+2] i_k[m-2])$$

Substituting the value of the function $\mathrm{U_k[n]}$ from (28) into (29), we obtain the following recurrence relation for the current $i_k[n]$:

$$i_k[n] = \frac{\frac{T}{3\lambda L_2}}{1 + \rho A \frac{T}{3\lambda L_2}} \left\{ B_2[n] + \sum_{m=1}^{n} (W_1[m] U_k[n-m] + 4W_1[n-m+1] U_k[m-1] + \right.$$
$$\left. +W_1[n-m+2] U_k[m-2]) \right\} \tag{30}$$

Therefore, having determined the value of the lattice functions $U_H[n]$, $i_k[n]$, a transition is made to find the changes in current and voltage at any point in the original system using recurrence relations (10), (11).

4. Conclusion

The results are of great practical importance for calculating the parameters of transmitting, receiving, measuring and other electrical devices that are used in electrical circuits with distributed parameters.

The developed numerical method can be developed to solve the problems of dynamics in electric circuits with distributed parameters containing elements with time-variable parameters, as well as nonlinear elements.

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