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Algorithm for determining the trajectory of maneuvers along the planned route on a geometric map of the terrain

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ABSTRACT

The paper considers the problem of calculating the trajectory of an unmanned aerial vehicle based on a geometric map, where obstacles and undulations of the terrain are specified in the form of spheres. A mathematical model for calculating the optimal trajectory is constructed, which, under additional smoothness conditions, is reduced to a boundary value problem for a system of nonlinear second-order differential equations. For practical purposes, a heuristic algorithm is proposed for the numerical calculation of one of the possible rational trajectories, which allows the drone to fly around obstacles with a minimum number of changes in the flight mode.

1. Introduction

By virtue of the intensive development of engineering and technology, unmanned aerial vehicles (drones) are now easily controlled by setting nodal points of the route. One of the objectives of automated control of the drone along a given route is to calculate the optimal trajectory. These points are usually entered by the operator via a tablet computer on the background of the map of the area. However, en-route flight between designated nodal points can be difficult due to obstacles and terrain. Namely, if an additional requirement is put forward for the drone's flight, for instance, to fly along the optimal length-wise trajectory, then the problem of its calculation arises. In the scientific and technical literature, there are many studies devoted to various variants of the problem of determining the trajectory to overcome obstacles, e.g. [1-4]. We are interested in the problem of determining a rational trajectory to overcome the elements of the terrain.

Information about the roughness of the relief is usually set in the form of a relief map in different formats, see [5-8] and others. However, extracting the necessary information from such sources requires additional costs for data maintenance and processing. In [9], a different representation of relief data was proposed, which significantly reduces the amount of stored information and simplifies its use when calculating the flight path. The specific characteristic of such a map, called a geometric map, is that obstacles and irregularities in the terrain are represented by simple geometric shapes, set by a limited number of parameters.

This paper considers the problem of calculating the trajectory of a drone based on such a map, where obstacles and irregularities of the terrain are specified in the form of spheres. An algorithm for

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calculating the trajectory is described, which allows an unmanned aerial vehicle to fly around obstacles with a minimum amount of flight mode change.

2. Problem statement

Although the coordinates of points above the ground are usually specified by indicating geographic latitude and longitude, given the relatively small flight distances of drones, it is possible to introduce a rectangular coordinate system and operate it when calculating the trajectory. Let us introduce a $Oxyz$ right-handed Cartesian coordinate system, directing the Oz axis vertically upward from the starting point O located on the ground. It is assumed that the flight will be carried out in some upper half-space

$$h_0 \leq z. \qquad (1)$$

where h_0 is a prescribed number.

Following [9], we will assume that the area is represented by a geometric map with elements of the "spatial point" type. For clarity, we will give an explanation about such elements..

The "spatial point" element with parameters $\{(x_i, y_i, z_i),\ r_i\}$ is a sphere

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \leq r_i^2, \qquad i = 1,2 \ldots. \qquad (2)$$

the upper part of which at (1) contains the elevation of the relief. Such an element makes it possible to separate the elevation in question or a certain part of it from the rest of the space with some margin of the spatial "layer" of safety (Fig. 1).

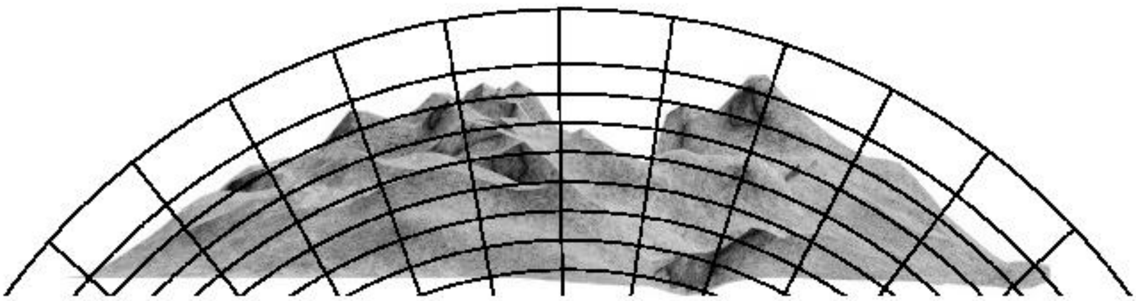

Fig. 1. The "spatial point" element covers a separate elevation with some margin of the "layer" of safety

When creating a geographic map of the area, these elements are generated by appropriate software based on a physical map of the area, specified, for instance, in the DTED (Digital Terrain Elevation Data) format [8].

As mentioned above, the flight route is entered by the operator via the program interface, indicating the nodal points on the background of the terrain map, thereby these points are tied to points on the earth's surface. We denote these nodal points by $M_k(x_{Mk}, y_{Mk}),\ k = 1,2, \ldots$, which are given in terms of the coordinates of the Oxy plane. Geometrically, the points $M_k(x_{Mk}, y_{Mk})$ represent the projections of the nodal points of the planned flight route on Oxy. Obviously, the calculated trajectory should be described by all coordinates x, y, z. It is assumed that the drone first occupies some starting point $M_0(x_0, y_0, z_0)$, then follows the given route. The height z_0 ($h_0 \leq z_0$) is considered known. In principle, the trajectory itself can also be represented as a sequence of the points $\{(x_i, y_i, z_i)\}$. In this case, if the relief of the terrain is not ignored, their projection on Oxy, generally speaking will not coincide with the nodal points M_k. In particular, the end point M_k of the route segment $M_{k-1}M_k$ may not be reachable, i.e. it does not satisfy at least one of the inequalities (2).

Therefore, when calculating the trajectory of the drone's movement, first it is necessary for each route segment to reasonably determine the "reachable" nodal points. If necessary, replacing the nodal points of the route M_k with the corresponding points to be reached and denoting them again by M_k, in the future, to determine the entire trajectory, a single calculation algorithm can be applied in all separate segments $M_{k-1}M_k$, $k = 0,1,2, \dots$. Automating the determination of the coordinates of the "reachable" nodal points is a separate task. It is not considered in the framework of the presented study and it is assumed that all specified nodal points M_k are "reachable".

Thus, the given route is the initial reference for calculating the flight path. Obviously, there can be infinitely many such trajectories.

The objective is to calculate the flight trajectory along a given route, such that it connects the nodal points and its length is the smallest. It should also remain within the permissible height limits ($h_0 \le z$) and not intersect with elements (2).

3. Mathematical formalization of the problem

Thus, the flight route is set in the form of a sequence of nodal points M_k, ($k = 1,2, \dots$) and information on the terrain relief in the form of a finite number of "spatial point" elements:

$$(x - x_{Bi})^2 + (y - y_{Bi})^2 + (z - z_{Bi})^2 \le r_{Bi}^2 , \quad i = 1, 2,3, \dots,$$

where $x_{Bi}, y_{Bi}, z_{Bi}, r_{Bi}$ are known numbers. The set x_{Bi}, y_{Bi}, z_{Bi} represents the coordinates of the base point (or center) of the i-th element:

$$B_i(x_{Bi}, y_{Bi}, z_{Bi}).$$

Consider a route segment $M_{k-1}M_k$, ($k = 1,2, \dots$). We successively apply the following transformations to the nodal points M_{k-1}, M_k and to all base points B_i:

- A_μ – parallel translation of coordinates onto the vector μ,
- A_α – compression of the segment length by α times,
- $A_\varphi = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$ – rotation around the Oz axis by the angle φ,

where $\mu = \left(-x_{M_{k-1}}, -y_{M_{k-1}}, -z_{M_{k-1}}\right)$,

$$\alpha = \sqrt{\left(x_{M_k} - x_{M_{k-1}}\right)^2 + \left(y_{M_k} - y_{M_{k-1}}\right)^2 + \left(z_{M_k} - z_{M_{k-1}}\right)^2},$$

$$\varphi = \arccos\left[\alpha^{-1}\left(x_{M_k} - x_{M_{k-1}}\right)\right].$$

Then the spatial segment $M_{k-1}M_k$ will be mapped onto the segment $[0, 1]$ of the Ox axis. Suppose that with this transformation, the nodal point M_k is mapped to the point $\breve{M}_k = (0,0, \tilde{z}_M)$, and the points B_i to the points $\breve{B}_i(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$. Replacing the radius of elements (2), respectively, with $r_i \equiv \alpha r_{Bi}$, we will construct a mathematical model of the optimal trajectory relative to the segment $[0, 1]$.

Let us denote by $y(x)$ and $z(x)$ the functions representing the desired trajectory $x \in [0, 1]$. We will assume that the functions $y(x)$ and $z(x)$ are sufficiently smooth. The requirement that the trajectory length be minimal can be written in the form of minimizing the following functional [10]:

$$\int_0^1 F\left(y'(x), z'(x)\right) dx, \tag{3}$$

where $F\big(y'(x), z'(x)\big) = \sqrt{1 + \big(y'(x)\big)^2 + \big(z'(x)\big)^2}$. The trajectory under consideration must connect the origin to the nodal point $\breve{M}_k$, i.e.

$$\begin{cases} y(0) = 0, & z(0) = 0, \\ y(1) = 0, & z(1) = \tilde{z}_M. \end{cases} \tag{4}$$

Constraint (1) will be written as

$$h_0 \le z(x). \tag{5}$$

The conditions for bypassing elements (2) can be written as

$$f_i(x, y(x), z(x)) \equiv r_i^2 - (x - \tilde{x}_i)^2 - (y(x) - \tilde{y}_i)^2 - (z(x) - \tilde{z}_i)^2 \ge 0, \;\; i = 1, 2,3, \dots . \tag{6}$$

Thus, the problem of determining the optimal trajectory can be reduced to the problem of minimizing functional (3) under conditions (4)-(6).

Obviously, problem (3)-(6) can be investigated by the methods of the calculus of variations or the theory of optimal control [11, 12]. Introducing the Lagrange multipliers (coefficients) λ_j, $j = 0, 1, 2, \dots, n$, and applying the Kuhn-Tucker theorem, problem (3)-(6) can be rewritten as the problem of minimizing the functional

$$\int_0^1 \left[F\big(y'(x), z'(x)\big) + \sum_{j=1}^{n} \lambda_j f_i(x, y(x), z(x)) + \lambda_0 (z(x) - h_0) \right] dx.$$

Setting forth additional conditions for the smoothness of $y(x)$ and $z(x)$, we can reduce it to a boundary value problem for the system of Euler equations (necessary extremum condition):

$$\begin{cases} \dfrac{y''(1 + (z')^2) - z''z'y'}{2(1 + (y')^2 + (z')^2)^{3/2}} + \displaystyle\sum_{j=1}^{n} \lambda_j \big(y - \tilde{y}_j\big) = 0, \\ \dfrac{z''(1 + (y')^2) - y''y'z'}{2(1 + (y')^2 + (z')^2)^{3/2}} + \displaystyle\sum_{j=1}^{n} \lambda_j \big(z - \tilde{z}_j\big) - \lambda_0 = 0. \end{cases} \tag{7}$$

Being a nonlinear problem, solving problem (6)-(4) requires the use of more complex numerical methods (e.g., [13]).

Therefore, in practice, the preference, whenever possible, is given to simplified methods for determining the trajectory, which is considered rational in a sense. We call a trajectory rational if its further direction from any current point to the point of change is such that it realizes the greatest decrease in the distance to the final point of the route.

Taking into account that the constraints causing the relief obstacles are spherical surfaces, and short lines on the set of spherical surfaces are geodesic lines or their intersection lines, we chose below to look for some rational trajectory using the heuristic algorithm instead of the optimal trajectory.

4. Formalization of a numerical problem for calculating a rational trajectory

First, we formulate the numerical problem of calculating the trajectory. Let us introduce on the segment $[0, 1]$ the sequence of nodal points $\{x_j = jh, \; j = 0, 1,2, \dots, N\}$, where N is some natural number, $h = \frac{1}{N}$. Let us denote by y_j and z_j the coordinates of the trajectory points along Oy and Oz, corresponding to x_j, $j = 0, 1,2, \dots, N$. Numerical calculation of the optimal trajectory implies finding such numbers y_j and z_j that

$$z_j \geq h_0, \qquad (x_j - \tilde{x}_i)^2 + (y_j - \tilde{y}_i)^2 + (z_j - \tilde{z}_i)^2 \geq r_i^2, \qquad j = 0,1,2,\dots,N, \;\; i = 1,2,3,\dots \quad (8)$$

$$y_0 = y_N = z_0 = 0, \quad z_N = \tilde{z}_M,$$

$$\sum_{j=1}^{N} \sqrt{h^2 + (y_{j-1} - y_j)^2 + (z_{j-1} - z_j)^2} \rightarrow \min \quad (9)$$

The following concept of calculating a rational trajectory is proposed.

We will "build" the desired trajectory iteratively. As an initial approximation, we take a line segment that connects the starting point $(0,0,0)$ with the end point $(1,0,\tilde{z}_M)$. If this segment is not intersected by relief elements, then it will be the optimal trajectory. In this case, the coordinates of the nodal points could be calculated at $j = 0$ from the equation of the line containing the given segment:

$$\frac{x-1}{x_j - 1} = \frac{y}{y_j} = \frac{z - \tilde{z}_M}{z_j - \tilde{z}_M}. \quad (10)$$

(A). ***First iteration***. Moving along line (10), we find the first pair of successive points $(x_{j-1}, y_{j-1}, z_{j-1})$ and (x_j, y_j, z_j) such that for the first, conditions (8) are satisfied, but for the second one, they are not fulfilled. In this case, we will look for a point (x_j, y, z) such that conditions (8) are satisfied for it, and the following sum is minimal:

$$S_j(y,z) \equiv \sqrt{h^2 + (y - y_{j-1})^2 + (z - z_{j-1})^2} + \sqrt{(1 - x_{j-1})^2 + y^2 + (z - \tilde{z}_M)^2} \quad (11)$$

In general, such a point may not be the only one. Let us choose one of them and take it as a new nodal point (x_j, y_j, z_j). The method for determining this solution is given below in Section 5.

Let us consider the line segment connecting it to the end point $(1, 0, \tilde{z}_M)$, the equation of which has form (10). If it does not intersect by the relief elements, we consider the first iteration complemented. In this case, the coordinates of the subsequent nodal points will be calculated from equation (10). Otherwise, repeat procedure (A).

Procedure (A) is repeated until the end nodal point $(1, 0, \tilde{z}_M)$ is reached, thereby completing the first iteration. The trajectory obtained after the completion of procedure (A) will be called the base trajectory. By the end of the first iteration, the value of sum (9) is calculated.

(B). ***Next iteration***. The nodal point j (starting with $j = 0$) is considered for the reachability of the nodal points $j + 1, j + 2, \dots$ by a direct connection, provided that the points of this line satisfy conditions (8). Suppose the point $j + k$ $(j + k \leq N)$ is such a point. Then the coordinates of the nodal points $j + 1, j + 2, \dots, j + k - 1$ are replaced with new ones, which are found from the equation of the line connecting the points (x_j, y_j, z_j) and $(x_{j+k}, y_{j+k}, z_{j+k})$. This procedure moves the nodal points found in the previous iteration and located on some "curved" nodal points to the line segments closing them. If nodal point $j + k$ is not the end point of the trajectory, i.e. if $j + k < N$, then procedure (B) is applied further from this point until the nodal point $(1, 0, \tilde{z}_M)$ is reached.

The new value of sum (9) is calculated. If the new value of this sum is not less than its previous value, then the search for the rational trajectory ends. Otherwise, repeat procedure (B).

Fig. 2 shows the stages of iterative construction of a rational trajectory connecting the nodal points M_1 and M_2: the result of the first iteration is shown with a thin line; the result of the second iteration is shown with a bold line. Part of the bold line coincides with the thin line. Ellipses are projections of geodesic lines on spheres onto the Oxy plane.

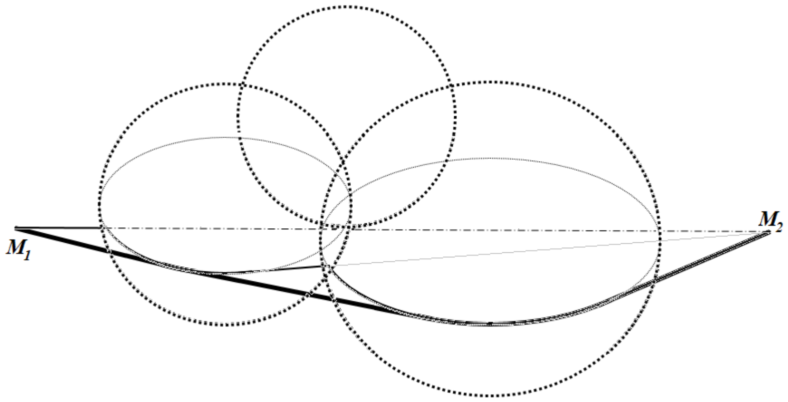

Fig. 2. Iterative construction of a rational trajectory connecting the nodal points M_1 and M_2.

5. Method of implementation of the first iteration

The search for the next nodal point in the construction of the first iteration is carried out among the points (x_j, y, z) for which sum (11) reaches a minimum. To this end, we can proceed as follows.

Find the intersection point of the segment connecting the points $(0,0,0)$ and $(1,0,\tilde{z}_M)$ with the plane $\{x = x_j\}$. The coordinates of the intersection point will be:

$$\begin{cases} x^* = x_j, \\ y^* = \dfrac{x_j - 1}{x_{j-1} - 1} y_{j-1}, \\ z^* = \dfrac{x_j - 1}{x_{j-1} - 1}\left(z_{j-1} - \tilde{z}_M\right). \end{cases}$$

Denote by $I(y^*)$ set of numbers i of only those elements (2), for which the condition

$$D_i(y^*) \equiv r_i^2 - \left(x_j - x_i\right)^2 - (y^* - y_i)^2 \geq 0$$

is satisfied, and calculate $z_{max}(y^*) = \max_{i \in I}\left\{z_i + \sqrt{D_i(y^*)}\right\}$.

Next, we repeat the construction of the set $I(y_k^*)$ for the points $y_k^* = y^* + k \cdot \Delta y$, $k = \pm 1, \pm 2, \ldots$ located at the nodes of a regular grid with some allowable step Δy within the set $\left\{y^* - (z_{max}(y^*) - z^*) \leq y \leq y^* + (z_{max}(y^*) - z^*), x = x_j\right\}$ and calculate the corresponding values $z_{max}(y_k^*) = \max_{i \in I}\left\{z_i + \sqrt{D_i(y_k^*)}\right\}$.

As the coordinates y_j and z_j corresponding to $x = x_j$, such values of y_k^* and $z_{max}(y_k^*)$ are selected for which the minimum of the function $\sqrt{(k \cdot \Delta h)^2 + (z^* - z_{max}(y_k^*))^2}$ by $k = 0, \pm 1, \pm 2, \ldots$.

As mentioned above, y_j, z_j may not be the only one. In this case, the selection is made on the basis of additional constraints, for instance, the requirement of minimality of z or y.

It should be noted that it is always possible to give an example so that the trajectory constructed in accordance with the proposed heuristic algorithm is not optimal. However, in most practical cases, such a trajectory turns out to be acceptable.

6. Conclusion

The paper considers the problem of calculating the trajectory of an unmanned aerial vehicle based on a geometrical map of the terrain, where obstacles and terrain irregularities are specified in the form of spheres. The problem of determining the optimal trajectory in accordance with the given route of movement is formulated. A heuristic algorithm is proposed for numerical calculation of one of the possible rational trajectories.

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