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The problem of optimal location of the military logistics center

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ABSTRACT

The article considers the problem of determining the optimal location of the logistics center in the area. To solve the problem, it is proposed to place the logistics center at one of the nodes of the transport infrastructure. A mathematical model is given to determine the economically viable location of the logistics center on the basis of hypothetical information about the coordinates of the terminal where the logistical resources are received in the transport infrastructure and the military units to be serviced. Taking into account other important factors, the algorithm is described for calculating transportation costs for each case where the center is located at the node and their ranking in ascending order in order to accurately determine the location of the logistics center (assuming the annual recurring costs do not change).

1. Introduction

The efficiency of the logistics center in comparison with other production and service facilities is determined by its geographical location. For this reason, when creating a logistics center, special attention is paid to its location in a convenient geographical area and its proximity to the points where it will operate (carry out transportation). In general, the location of the facility in favorable coordinates is considered one of the classic problems of optimization theory. The solution of such problems in the convex set is based on the calculation of the unconditional extremum of the function, which expresses a certain optimality criterion. An example of this is the "Weber problem" (a method of locating a warehouse that will service production facilities in a particular area) [1]. In this case, it is necessary to find a point indicating the location of the warehouse, so that the cost of transporting goods to a finite number of consumer enterprises located on that plane is minimal. According to the Weber problem, when determining the location of the warehouse, it should be taken into account that the cost of transporting a single load over the same distance is different for each consumer enterprise. In this regard, to solve the Weber problem, the location of the warehouse is parameterized (the rectangular coordinates entered in relation to the area are taken as a parameter). The objective function is constructed, which represents the sum (depending on the coordinates) of the costs of transportation to individual points (enterprises). The solution of the problem is calculated as the minimum of the

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objective function.

As the set (zone, area) in which the problem is considered becomes more complex, new approaches are needed. For instance, the set we consider in [2] consists of an area obtained by subtracting rectangular zones from the convex set. This type of research is often theoretical. Real placement problems are related to a specific geographical location. Both the warehouse and the consumer outlets are located at discrete points of the appropriate infrastructure. Therefore, the issue of discrete placement is considered on the graph [3]. Distance between stations and transportation costs are given in tabular form. When solving the discrete placement problem, one of the following approaches is traditionally used:

1. Determining the optimal location of the warehouse by calculating all possible options. In this case, the objective function of the total cost of transportation from the proposed location is constructed for the purpose of locating the warehouse. By calculating the values of this function for a finite number of different places, the most suitable of them is selected. This approach is used when the number of considered discrete locations is small and the objective function is simple.

2. Applying a heuristic approach to finding the optimal warehouse location when the objective function is complex. The essence of this approach is that, based on other logical considerations, a complete report is implemented for a small number of points, excluding inappropriate (unacceptable) options from the set of possible solutions.

3. If the characteristics of the objective function allow, solving the problem of determining the optimal warehouse location by the application of mathematical methods. In the simplest case, the "center of gravity method" is applied to solve the Weber problem, depending on the consumer needs and their distribution in the graph [4; 5; 6].

At present, various location problems are studied in the scientific and technical literature depending on the structure of the set of solutions considered, the characteristics of the transportation organization and other factors. A brief summary of a number of articles on the location of logistics facilities is given in [7].

Unlike enterprises of a wide range of activities, the location of the military logistics center's points and the military units it services are known in advance and do not change over the years. This can solve the problem of optimal choice of location in the geographical area by ensuring the efficiency of its operation. The main requirements for the establishment of a military logistics center and the mechanism for assessing its military-economic efficiency are given in [8]. In this article, it is clear from the analysis of the costs incurred during the operation of the military logistics center that the structure of the center's operating expenses is approximately the same over the years, and among these expenses, it is mainly the following that depend on the geographical location:

- Q_1 – expenses related to the delivery of logistical resources in use from the central warehouse to the logistics center;
- Q_2 – transportation costs of repair crews for the maintenance of automotive and armored vehicles.

Therefore, the minimization of the sum of these quantities should be especially taken into account in determining the optimal location of the logistics center.

We assume that the logistics center being built should be close to the infrastructure in the area. If necessary, the construction of a new road should provide access to its infrastructure, and the distances between the points should be known. Based on these factors, to determine the optimal location of the logistics center, it is necessary to write an appropriate analogue of the Weber problem.

Within the conditions mentioned in the presented article, a mathematical model of the problem of locating a military logistics center is constructed and the algorithm of its numerical solution is given.

2. Formulation of the problem of locating the military logistics center

As a rule, the effective organization of the logistics center depends on its location in the existing transport infrastructure. As mentioned above, if necessary, access to the existing infrastructure of the logistics center being created can be provided by the construction of a new road. However, the road implementing this access must be many times shorter than the total length of the route covered during transportation. In this regard, the length of the access road to the infrastructure may not be taken into account when determining the optimal location of the logistics center. In other words, the location of the logistics center can be considered practically in the existing infrastructure.

It should also be noted that restrictions related to security and protection (the logistics center being 30–40 km away from the enemy artillery near the line of contact), as well as to the geography of the area (dense forests, rivers, lakes, valleys, mountain ranges, etc.), may be imposed when determining the location of the military logistics center [9].

Taking into account these restrictions, possible areas (sets) for the location of the logistics center are identified. For example, in Fig. 1, the possible location areas related to these restrictions are indicated with a bold line. Here, military units are denoted by Y_1, Y_2, Y_3, Y_4, transport terminal receiving material and spare parts by Y_0, crossroads by $Y_5, \ldots, Y_{12}$. Sections Y_5Y_6, Y_8Y_9, Y_9Y_{10}, Y_9Y_{11} and Y_9Y_{12} of the roads shown in the figure are considered to be possible areas for the location of the logistics center.

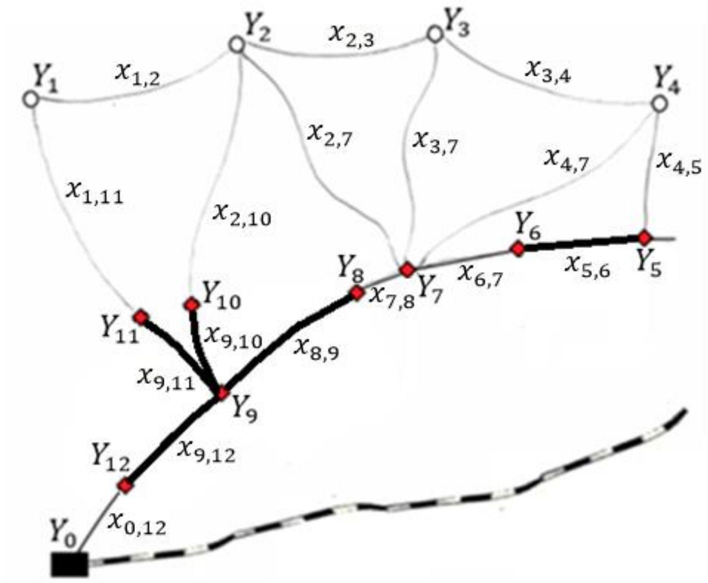

Fig. 1. The layout of the location of a logistics center in the arbitrary area

As mentioned above, after the establishment of the military logistics center, the structure of its operating expenses is be approximately the same over the years. Depending on the distance between the points, the formulas for calculating the mentioned costs Q_1 and Q_2 can be presented as follows.

Let's denote the number of military units serviced by the logistics center by n and number the military units as $s = 1,2, \ldots n$. In accordance with [8], the formula for calculating the cost of delivery of property and spare parts available in the central warehouse from the transport terminal to the logistics center:

$$Q_1(l_0) = a_0 l_0 + b, \tag{1}$$

and the formula for calculating the cost of delivery of maintenance crews from the logistics center to military units can be written in the following form:

$$Q_2(b_1, b_2, \dots b_n) = \sum_{s=1}^{n} a_s \cdot l_s \quad (2)$$

In these formulas, l_0 is the distance from the transport terminal to the warehouse of the logistics center, l_s $(s = 1,2, \dots n)$ is the length of the shortest route from the logistics center to the s-th military unit. The coefficients $a_0, \dots, a_n, b$ are positive quantities that do not depend on the length of the route covered. These coefficients are calculated on the basis of the indicators reflected in the annual plan of operation and maintenance of equipment serviced by the logistics center. The calculation procedure is given in [8].

As a rule, the distances between the points of service of the logistics center (locations of transport terminal and military units), as well as between the main nodes of the infrastructure are given in tabular form. In general, we will denote these points by Y_m $(m = 1, 2, \dots, m_0)$, where m_0 is the total number of points. Let us assume that among the points included in the table, there are also nodes that determine the possible areas for the location of the logistics center. If the table of initial distances in the infrastructure described in Fig. 1 represents only the distances between points $Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_7, Y_9$, the table of distances between them should cover all nodes in the figure, including the nodes that determine possible areas for the location of the logistics center: $Y_0, Y_1, Y_2, \dots, Y_{12}$.

The issue of determining the optimal location of the military logistics center means finding as many locations as possible so that the planned annual transportation costs are minimal. Thus, the problem of determining the location of the military logistics center can be formulated as finding the point that minimizes the sum total

$$Q = Q_1 + Q_2 \quad (3)$$

in the given set on the infrastructure.

In addition to the assessment of economic indicators, other factors are taken into account when determining the location of the military logistics center. In this regard, the economically efficient options should be presented to the decision-making party in ascending order.

As can be seen from function (3), the quantity b does not play a role in finding its minimum point, so we will assume that $b = 0$.

3. Mathematical model and solution algorithm of the optimization problem

Determining the optimal location for the logistics center within each separately considered area can be investigated as a separate problem.

For example, the problem of determining the optimal location in each of the areas Y_5Y_6, Y_8Y_9, Y_9Y_{10}, Y_9Y_{11} and Y_9Y_{12} in Fig. 1 can be considered independently. In this case, in determining the optimal location of the logistics center, it is assumed that the length of the route between each of the points $Y_5, Y_6, Y_8, Y_9, \dots, Y_{12}$ and points $Y_0, \dots, Y_4$ is known.

Thus, let us assume that the length of the routes between the points is known. Taking into account the difficulty of the road, the distance can be measured in *km* or other units, e.g. hours. Suppose $x_{i,j}$ denotes the length of the route between point i and point j. For simplicity, suppose that $x_{i,j} = x_{j,i}$. Note that if this condition is not met, the following considerations remain in force. If there is no road from point i to point j, the value of $x_{i.j}$ can be considered a great number. For instance, for the problem under consideration, $x_{i,j} = 10000$ *km* can be taken as the absence of a road.

The distance between the roads can be given in the matrix form as follows:

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,m_0} \\ \cdots & \cdots & \cdots \\ x_{m_0,1} & \cdots & x_{m_0,m_0} \end{bmatrix}.$$

For the points given in Fig. 1, the matrix X is 13×13 ($m_0 = 13$), and its elements can be written, e.g., as in Table 1. In this table, the arbitrary numbers of the points are given from 0 to 12 in the 1st row and in the 1st column.

Let us denote the shortest route from point i to point j by $z_{i,j}$. There are various algorithms to calculate these distances [10]. According to the example, the distances between other points are given in gray cells on the basis of known distances between adjacent points (indicated in bold in white cells in the table). Note that the shortest route from point i to point j may not be $x_{i,j}$. This may be due to the fact that the route between the points does not coincide with the geodetic line.

Table 1

Distances between the points

	0	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***
0		53	40	58	61	65	51	39	30	15	21	23	**5**
1	53		**25**	43	67	71	52	45	54	38	44	**30**	48
2	40	**25**		**18**	42	46	32	**20**	29	25	**19**	33	35
3	58	43	**18**		**24**	35	33	**21**	30	43	37	51	53
4	61	67	42	**24**		**11**	25	**23**	32	47	53	55	57
5	65	71	46	35	**11**		**14**	26	35	50	56	58	60
6	51	52	32	33	25	**14**		**12**	21	36	42	44	46
7	39	45	**20**	**21**	**23**	26	**12**		**9**	24	30	32	34
8	30	54	29	30	32	35	21	**9**		**15**	21	23	25
9	15	38	25	43	47	50	36	24	**15**		**6**	**8**	**10**
10	21	44	**19**	37	53	56	42	30	21	**6**		14	16
11	23	**30**	33	51	55	58	44	32	23	**8**	14		18
12	**5**	48	35	53	57	60	46	34	25	**10**	16	18	

Let us denote the separately considered location areas by $\Omega_1, \Omega_2, \dots, \Omega_{\mathrm{k}_0}$, where k_0 is the total number of possible areas. Since each area Ω_k ($k = 1, 2, \dots, \mathrm{k}_0$) is geometrically a part of a one-dimensional curve, its points can be identified depending on one parameter (variable), let us denote this parameter by x. Suppose that x is the natural parameter of the curve – the distance from the selected end point of the area to the point under consideration [11]. If we denote the length of the section Ω_k by ω_k, the parameter x varies within the range $[0, \omega_k]$. The values $x = 0$ and $x = \omega_k$ correspond to the nodes of that part, which we will denote by $\hat{Y}_k^{(1)}$ and $\hat{Y}_k^{(2)}$, respectively. Let us denote the optimal length of the route from the point $\hat{Y}_k^{(i)}$, $(i = 1,2)$ to the transport terminal $s = 0$ and to the s-th military unit $s = 1,2, \dots n$ by $l_{k,s}^{(i)}$ $(i = 1,2)$. These distances are determined based on the corresponding $z_{k,s}$.

Then the optimal length of the route from each point corresponding to the value $x \in [0, \omega_k]$ to point s is calculated as follows:

$$l_{k.s}(x) = \min\left\{l_{k,s}^{(1)} + x, \omega_k + l_{k,s}^{(2)} - x\right\}. \tag{4}$$

An object moving from the point corresponding to the value $x \in [0, \omega_k]$ first arrives at the point $\hat{Y}_k^{(1)}$ or $\hat{Y}_k^{(2)}$, and from there travels to the s-th point along the route indicated in the table of distances. Note that in this case, there is no conflict in formula (4) when the object that arrived at the point $\hat{Y}_k^{(i)}$ continues its movement in the opposite direction and comes to the point $\hat{Y}_k^{(1-i)}$.

Based on formula (4), we need to find the value $x = \hat{x}_k$ minimizing the function

$$Q_{(k)}(a_1, a_1, \dots a_n, x) = \sum_{s=0}^{n} a_s \cdot l_{k.s}(x). \tag{5}$$

in order to determine the possible optimal location in the area Ω_k for each $k = 1, 2, \dots, \mathrm{k}_0$:

$$\hat{x}_k = \arg\min_x Q_{(k)}(a_1, a_1, \dots a_n, x).$$

On the other hand, since $a_1, a_1, \dots a_n$ are not negative quantities, it is easy to see that each function $l_{k.s}(x)$ is a concave function of the variable $x \in [0, \omega_k]$. As a result, function (5) will also be a concave function. Therefore, the function $Q_{(k)}(a_1, a_1, \dots a_n, x)$ can get its minimum at the end points of the defined interval, in other words, , $\hat{x}_k = 0$ or $\hat{x}_k = \omega_k$. When $Q_{(k)}(a_1, a_1, \dots a_n, x)$ is a constant function, all points of the interval $\Omega_{\hat{k}}$ will be equivalent. Hence, to find the optimal location, it is enough to look only at the end points of the interval $[0, \omega_k]$. Denoting $\hat{Q}_{(k)} = \min_x Q_{(k)}(a_1, a_1, \dots a_n, x)$, $(k = 1, 2, \dots, \mathrm{k}_0)$, we get

$$\hat{Q}_{(k)} = \min\{Q_{(k)}(a_1, a_1, \dots a_n, 0), Q_{(k)}(a_1, a_1, \dots a_n, \omega_k)\}.$$

The optimal location of the logistics center sought in the set of the considered areas can be determined from the minimum of $\hat{Q}_{(k)}$ numbers:

$$\hat{k} = \arg\min_{k=1,2,\dots,\mathrm{k}_0}\{\hat{Q}_{(k)}\},$$

$$x = \hat{x}_{\hat{k}} \in \Omega_{\hat{k}}.$$

Summarizing the above, the algorithm for solving the problem of determining the optimal location of the military logistics center can be given as follows:

1. Identifying the end points of possible locations (road section) determined in accordance with safety and security requirements.
2. Calculating the length of the shortest route from the considered points to the transport terminal and to each military unit.
3. Calculating the value of function (3) for each point.
4. Arranging the calculated Q values in ascending order and formatting the results to present to the decision-maker.

4. Conclusion

To solve the problem of determining the optimal location of the logistics center in the area, it is enough to study only the given nodes on the transport infrastructure. It has been proven that the economically efficient space is at one of these nodes.

Based on the proposed algorithm, it is possible to calculate the annual recurring costs at each node where the logistics center can be located and rank them in ascending order. This allows providing accurate information to the decision-maker, taking into account other factors when determining the location of the military logistics center.

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