On the problem of optimizing emissions of industrial enterprises

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ABSTRACT

The problem of optimization of emissions of industrial enterprises is solved, taking into account the minimum dose of pollution of environmentally significant zones. Economic costs for reorganization of technology of the considered industrial objects are taken as the main criterion so that the total economic costs for the given reduction of pollution would be minimal for the enterprises for the whole region. As a result, the problem is reduced to a linear programming problem using solution of basic and conjugate equations of transport and diffusion of polluting aerosols.

1. Introduction

The problem of environmental protection and rehabilitation is becoming one of the most pressing problems of science. The intensive development of industry and the associated process of increasing industrial emissions that pollute the environment is becoming very tangible for the ecological balance of our planet and its individual regions.

Literature addresses various aspects of the problem associated with the building of new industrial facilities that emit harmful aerosols into the atmosphere, taking into account the minimum pollution of nearby settlements, recreational areas, agricultural land and other environmentally significant facilities [1-5]. In the present study, we will consider another aspect of the problem. We will assume that all industrial enterprises in the area already exist and emit a given amount of harmful aerosols into the atmosphere. The problem is to determine for each enterprise such a permissible amount of emitted aerosols that their sum does not exceed sanitary permissible norms. At the same time, it is impossible to significantly underestimate the total emissions, because it will lead to a decrease in the economic performance of industrial facilities. Thus, we will be talking about such limits on emissions, which will still provide the maximum economic effect under the given restrictions.

2. Problem statement

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Suppose that in a given region G with boundary S at points $\boldsymbol{r}_i (i = 1,2,\dots,n)$ there are n industrial facilities A_i, emitting every second $Q_i (i = 1,2,\dots,n)$ aerosols, whose composition for simplicity will be assumed to be identical (Fig. 1). In the region G we select m ecological zones $G_k (k = 1,2,\dots,m)$, for each of which the maximum permissible concentrations of aerosol falling in the time interval $[0, T]$ are prescribed. As a result, we arrive at the following mathematical statement of the problem.

An equation for the diffusion of substances from n industrial facilities is given:

$$\frac{\partial \varphi}{\partial t} + div\, \boldsymbol{u}\varphi + \sigma\varphi = \frac{\partial}{\partial z} \nu \frac{\partial \varphi}{\partial z} + \mu\Delta\varphi + \sum_{i=1}^{n} Q_i \delta(\boldsymbol{r} - \boldsymbol{r}_i) \qquad (1)$$

under the conditions that

$$\varphi = f_S \quad \text{on} \quad \Sigma,$$
$$\frac{\partial \varphi}{\partial z} = \alpha\varphi \quad \text{on} \quad \Sigma_0, \qquad (2)$$
$$\frac{\partial \varphi}{\partial z} = 0 \quad \text{on} \quad \Sigma_{\mathrm{H}}.$$

Considering the problem (1), (2) to be climatically periodic (with a period equal to a year), we obtain the input data

$$\varphi(\boldsymbol{r}, T) = \varphi(\boldsymbol{r}, 0). \qquad (3)$$

Here the components of the wind velocity vector $\boldsymbol{u}$ are connected at each moment of time by the continuity relation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

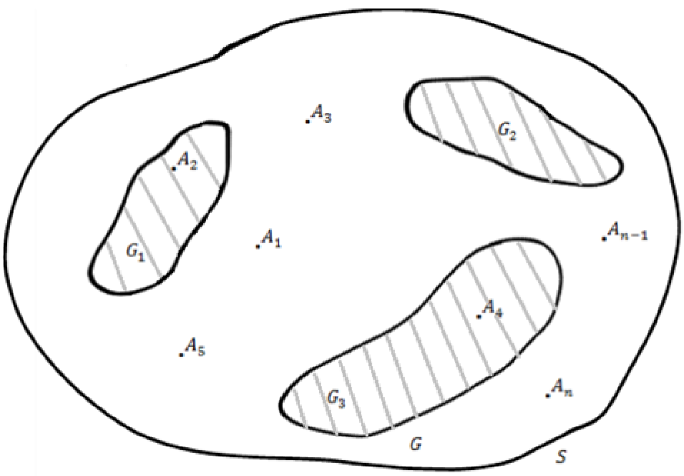

Fig. 1. The region where the industrial facilities are located

under the conditions that $w = 0$ at $z = 0, z = z_H$; ν, μ are coefficients of vertical and horizontal turbulent exchange, $\boldsymbol{r}_i = (x_i, y_i, z_i)$. The coefficient α describes the probability of an aerosol substance that fell to the ground surface reentering the atmosphere, and f_S is aerosol sources on Σ.

3. Solution

Consider the functional

$$Y_k = \int_0^T dt \int_{G_r} p_c \varphi dG, \tag{4}$$

which characterizes the sanitary dose of aerosols deposited on the ground surface $(z = 0)$ in the area of the ecological zone G_k. The task is to find such a set of planned aerosol emissions Q_i, which would ensure the average annual maximum permissible doses of aerosol pollution

$$Y_k \leq c_k, \quad k = 1,2, \dots, m, \tag{5}$$

with minimum economic costs for the technological reconstruction of enterprises, ensuring the specified output with a specified emission reduction [6].

Naturally, in this problem along with constraints (5) it is necessary to introduce a minimizing functional. As such we take

$$I = \sum_{i=1}^{n} \xi_i (\bar{Q}_i - Q_i), \tag{6}$$

where $\bar{Q}_i$ is the original and Q_i is the planned emission capacity, the coefficient ξ_i determines the capital investment in the technology that allows the output of the same volume of products while reducing emissions (per unit of emission capacity). Then, the functional I represents the full costs needed to improve the technology of all plants A_i when moving from emissions $\bar{Q}_i$ to planned emissions Q_i. As a result, we arrive at the problem of finding in (1)-(3) such emissions Q_i, that the following conditions are satisfied

$$I = \sum_{i=1}^{n} \xi_i (\bar{Q}_i - Q_i) = min,$$

$$Y_k \leq c_k, \quad k = 1,2, \dots, m. \tag{7}$$

Problem (1)-(3), (7) can be reduced to a linear programming problem [7]. In this case two different approaches are possible, one of which is implementable with basic equations, and the other with conjugate equations [8].

3.1. Optimization with basic equations

We represent the solution of problem (1)-(3) as a superposition of solutions of elementary problems [9]. Suppose that

$$\varphi = \sum_{i=1}^{n} Q_i \varphi_i(\boldsymbol{r}, t) + \varphi_S, \tag{8}$$

where $\varphi_i(\boldsymbol{r}, t)$ is the solution of the problem

$$\frac{\partial \varphi_i}{\partial t} + u \frac{\partial \varphi_i}{\partial x} + v \frac{\partial \varphi_i}{\partial y} + w \frac{\partial \varphi_i}{\partial z} + \sigma \varphi_i = \frac{\partial}{\partial z} v \frac{\partial \varphi_i}{\partial z} + \mu \Delta \varphi_i + \delta(\boldsymbol{r} - \boldsymbol{r}_i) \tag{9}$$

with the boundary conditions

$$\varphi_i = 0 \quad \text{on} \quad \Sigma,$$
$$\frac{\partial \varphi_i}{\partial z} = \alpha \varphi_i \quad \text{on} \quad \Sigma_0, \tag{10}$$
$$\frac{\partial \varphi_i}{\partial z} = 0 \quad \text{on} \quad \Sigma_{\text{H}}.$$

and the periodicity condition

$$\varphi_i(\boldsymbol{r}, T) = \varphi_i(\boldsymbol{r}, 0). \tag{11}$$

Along with problem (9)-(11) for $i = 1, 2, \dots, n$, we introduce another problem for determining the background aerosols arriving in the areas of G through the boundary S:

$$\frac{\partial \varphi_S}{\partial t} + div\, \boldsymbol{u}\varphi_S + \sigma\varphi_S = \frac{\partial}{\partial z}\nu\frac{\partial \varphi_S}{\partial z} + \mu\Delta\varphi_S \tag{12}$$

under the conditions that

$$\varphi_S = f \quad \text{on} \quad \Sigma,$$
$$\frac{\partial \varphi_S}{\partial z} = \alpha \varphi_S \quad \text{on} \quad \Sigma_0, \tag{13}$$
$$\frac{\partial \varphi_S}{\partial z} = 0 \quad \text{on} \quad \Sigma_{\text{H}},$$
$$\varphi_S(\boldsymbol{r}, T) = \varphi_S(\boldsymbol{r}, 0). \tag{14}$$

Suppose that each of problems (9)-(11) at $i = 1, 2, \dots, n$,, as well as problem (12)-(14) are solved. Then representation (8), which we now use to calculate the functionals Y_S, is valid. Indeed, by substituting (8) into (4), we obtain

$$Y_k = \sum_{i=1}^{n} Q_i a_{ik} + b_k, \tag{15}$$

where

$$a_{ik} = \int_0^T dt \int_{G_R} p_c \varphi_i(\boldsymbol{r}, t) dG,$$
$$b_k = \int_0^T dt \int_{G_R} p_c \varphi_S(\boldsymbol{r}, t) dG,$$
$$i = 1,2, \dots, n; \qquad k = 1,2, \dots, m.$$

Now a_{ik}, b_k are already known constants. Combining (7) and (15), we arrive at the problem

$$\sum_{i=1}^{n} \xi_i(\bar{Q}_i - Q_i) = min,$$
$$\sum_{i=1}^{n} Q_i a_{ik} + b_k \le c_k, \qquad k = 1,2, \dots, m. \tag{16}$$

It is convenient to proceed from Q_i to $q_i = \bar{Q}_i - Q_i \geq 0$.. Then we arrive at the linear programming problem of finding the optimal set q_i based on the solution of the problem

$$\sum_{i=1}^{n} \xi_i q_i = min,$$

$$\sum_{i=1}^{1} a_{ik} q_i \geq R_k, \qquad k = 1,2, \dots, m, \tag{17}$$

$$q_i \geq 0, \quad i = 1,2, \dots, n,$$

where $R_k = \sum_{i=1}^{n} a_{ik} \bar{Q}_i + b_k - c_k$.

Naturally, the number of restrictions may be increased by social and economic requirements arising from certain priority considerations.

3.2. Optimization with a conjugate problem

The conjugate problem is generated by Lagrange's identity [8]. Indeed, we introduce into consideration the operator

$$L = \frac{\partial}{\partial t} + div\,(u\,\cdot) - \frac{\partial}{\partial z} \nu \frac{\partial}{\partial z} - \mu\Delta + \sigma \tag{18}$$

on the set of functions $\varphi \in \Phi$, continuous together with their first-order derivatives with respect to t and second-order derivatives with respect to x, y. As for the derivative on z, continuity of $\nu\, \partial\varphi/\partial z$ is assumed. Further, suppose that each element of this set satisfies the conditions

$$\varphi = 0 \;\text{ on }\; \Sigma,$$

$$\frac{\partial\varphi}{\partial z} = \alpha\varphi \;\text{ on }\; \Sigma_0, \tag{19}$$

$$\frac{\partial\varphi}{\partial z} = 0 \;\text{ on }\; \Sigma_{\mathrm{H}},$$

$$\varphi(\boldsymbol{r}, T) = \varphi(\boldsymbol{r}, 0).$$

We introduce the scalar product

$$(g, h) = \int_0^T dt \int_G ghdG. \tag{20}$$

Now, in the notation adopted above, main problem (9)-(11) can be written in the operator form:

$$L\varphi_i = \delta(r - r_0). \tag{21}$$

We introduce the conjugate operator L^* using Lagrange's identity

$$\left(\varphi^*, L_\varphi\right) = (\varphi, L^*\varphi^*), \tag{22}$$

with φ and φ^* assumed to be real. Hence, by the standard method we arrive at the determination of the conjugate operator:

$$L^* = -\frac{\partial}{\partial t} - div\,(\boldsymbol{u}\,\cdot) - \frac{\partial}{\partial z} \nu \frac{\partial}{\partial z} - \mu\Delta + \sigma. \tag{23}$$

Each element $\varphi^* \in \Phi$, on which the operator L^*, acts has to be a continuous function having a first derivative with respect to t, a second derivative with respect to x, y, and a continuous conjugate flow $\nu\ \partial\varphi^*/\partial z$. In addition, φ^* must satisfy the conditions

$$\begin{gathered}\varphi^* = 0 \quad \text{on} \quad \Sigma, \\ \frac{\partial\varphi^*}{\partial z} = \alpha\varphi^* \quad \text{on} \quad \Sigma_0, \\ \frac{\partial\varphi^*}{\partial z} = 0 \quad \text{on} \quad \Sigma_{\rm H}, \\ \varphi^*(\boldsymbol{r}, T) = \varphi^*(\boldsymbol{r}, 0).\end{gathered} \tag{24}$$

Consider the problem conjugate to (21)

$$L^*\varphi_k^* = p_k, \tag{25}$$

where p_k is an as of yet undetermined function. Multiply (21) scalarly by φ_k^*, (25) by φ_i, and subtract one result from the other:

$$(\varphi_k^*, L\varphi_i) - (\varphi_i, L^*\varphi_k^*) = Q_i \int_0^T \varphi_k^*(r_i, t)dt - (\varphi_i, p_k). \tag{26}$$

Since $\varphi_i \in \Phi,\ \varphi_k^* \in \Phi^*$, taking into account (22), the left-hand side of (26) will turn to zero. Then

$$Y_{ik} = (\varphi_i, p_k) = Q_i \int_0^T \varphi_k^*(r_i, t)dt; \tag{27}$$

as p_k we take

$$p_k(r) = \begin{cases} b + a\delta(z) & \text{on} \quad G_k, \\ 0 & \text{outside} \quad G_k. \end{cases}$$

As a result, for the functional Y_{ik} we have two equivalent forms generated by equality (27) and elementary problems with unit source Q_i:

$$Y_{ik} = \int_0^T dt \int_\Omega p_k\varphi_i d\Sigma, \qquad Y_{ik} = Q_i \int_0^T \varphi_k^*(\boldsymbol{r}_i, t)dt. \tag{28}$$

Thus, Y_{ik} can be found as a result of solving either the main problem or the conjugate problem. Since the optimization problem using the basic equations is considered above, we focus on the conjugate problem (25), which we write down in the form:

$$-\frac{\partial\varphi_k^*}{\partial t} - div\,\boldsymbol{u}\varphi_k^* + \sigma\varphi_k^* = \frac{\partial}{\partial z}\nu\frac{\partial\varphi_k^*}{\partial z} + \mu\Delta\varphi_k^* + p_k(\boldsymbol{r}), \tag{29}$$

$$\begin{gathered}\varphi_k^* = 0 \quad \text{on} \quad \Sigma, \\ \frac{\partial\varphi_k^*}{\partial z} = \alpha\varphi_k^* \quad \text{on} \quad \Sigma_0, \\ \frac{\partial\varphi_k^*}{\partial z} = 0 \quad \text{on} \quad \Sigma_{\rm H},\end{gathered} \tag{30}$$

$$\varphi_k^*(\boldsymbol{r}, T) = \varphi^*(\boldsymbol{r}, 0). \tag{31}$$

If problem (29)-(31) is solved, we can find the functional Y_{ik} by simple integration. The further way of formulating the optimization problem is already clear.

Introduce the functional

$$Y_k = \sum_{i=1}^{n} Q_i \int_0^T dt \varphi_k^*(\boldsymbol{r}_i) + \int_0^T dt \int_{G_k} \varphi_S dG \tag{32}$$

and denote

$$a_{ik}^* = \int_0^T \varphi_k^*(\boldsymbol{r}_i) dt, \qquad b_k^* = \int_0^T dt \int_{G_k} \varphi_S dG. \tag{33}$$

Then (32) will be written in the form

$$Y_k = \sum_{i=1}^{n} a_{ik}^* Q_i + b_k^*. \tag{34}$$

Thus, similarly to the main problem case, we arrive at the optimization problem for conjugate equations:

$$\sum_{i=1}^{n} \xi_i (\bar{Q}_i - Q_i) = min,$$

$$\sum_{i=1}^{n} a_{ik}^* Q_i + b_k^* \le c_k, \qquad k = 1,2, \dots, m, \tag{35}$$

or, introducing $q_i = \bar{Q}_i - Q_i \ge 0$, we transform problem (35) into the following:

$$\sum_{i=1}^{n} \xi_i q_i = min,$$

$$\sum_{i=1}^{n} a_{ik}^* q_i \ge R_k^*, \qquad k = 1,2, \dots, m, \tag{36}$$

$$q_i \ge 0, \quad i = 1,2, \dots, n,$$

where $R_k^* = \sum_{i=1}^{n} a_{ik}^* \bar{Q} + b_k^* - c_k$. Thus, we arrive again at a linear programming problem.

4. Conclusion

In various cases it is convenient to formulate an optimization problem by solving either basic or conjugate equations [8, 10]. If the number of enterprises emitting aerosol into the atmosphere is small, and the number of environmentally significant zones is larger, it is more convenient to use basic equations. Otherwise, it is more convenient to use conjugate equations.

As for the numerical implementation of the algorithms, since the main and conjugate problems are linear and periodic in time, they can be solved by periodization, starting with some initial data and continuing the solution until the periodicity occurs [11]. Usually two or three year cycles of calculation are sufficient. It is important to note that the conjugate problem should be solved in the reverse time direction because, as noted earlier, in this case the validity of the problem will be observed in the calculation. As for linear programming problems, they are solved by standard methods [12, 13]. Since $q_i \ge 0$ and all coefficients a_{ik}, a_{ik}^* are also positive, the solution of the problem is found on the faces of the polyhedrons forming when constructing the region of constraints.

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