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A method for approximate determination of the indicators of development of a reservoir with rocks subjected to relaxation deformation in single-phase fluid flow

B.Z. Kazymov[1*], R.M. Zeynalov[2]

[1]*Oil and Gas Institute of ANAS, Baku, Azerbaijan*
[2]*Institute of Control Systems of Azerbaijan National Academy of Sciences, Baku, Azerbaijan*

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The proposed solution allows, with the flow rate of the well given, determining the wellbore and pattern values of reservoir porosity and the distribution of reservoir pressure across the reservoir, including estimating the pattern and wellbore pressures, from the indicators of development of oil deposits whose rocks are subjected to relaxation deformation.

1. Introduction

Rocks constituting oil and gas fields in the process of field development under conditions of high geostatic pressure are usually subjected to non-linear deformation of a relaxational and creeping nature. In practical cases in which these circumstances are expressed, there is a need to investigate the appropriately formulated problems related to the prediction of development indicators that allow the necessary engineering estimates in order to achieve effective development of the reserve potential of the fields. This also makes it possible to develop theoretical foundations for the development and operation of oil and gas fields located in conditions of such high geostatic pressure [1-4].

The problems of predicting indicators characterizing the process of development of oil and gas fields, the rocks of which are subject to inelastic deformation, involve, first of all, the formulation of problems of underground hydro-gas dynamics, expressed in the filtration of fluids and gases in reservoirs, and their solution using possible research methods [2, 4]. In this area, many calculation methods have been created so far, and at present, creation of corresponding new methods continues on the basis of these methods and on the problems of a different formulation than that of the problems they cover, as well as with the application of various solution methods [5-9].

For instance, in [5] the problem of developing gas-bearing formations whose rocks undergo relaxation deformation is considered in a simple formulation (unsteady flow into a single well),

[*]Corresponding author.
E-mail addresses: bunyadkazymov1969@gmail.com (B.Z.Kazymov), raminz.math@gmail.com (R.M.Zeynalov).

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approximate solutions are given to determine the reservoir development indicators corresponding to this case; in [6], [7] and [8] nonlinear elastic and inelastic deformations of rocks are considered, including the proposed solutions to problems of selecting the optimal mode of production wells and determining formation parameters when developing and operating gas and gas wells; and in [9] solutions of problems of determining the required number of wells, providing a given dynamics of production in the development of deep oil fields are given.

In view of the above, in the current study we propose an approximate solution of the problem in a hydrodynamic formulation, involving the determination of the indicators of development of oil fields, whose rocks are subject to relaxation deformation in a single-phase fluid flow, similarly to the approach used in [5].

2. Problem statement

Consider an axisymmetric plane-parallel radial flow of liquid (single-phase oil) in a well with a radius of r_q in a circular bounded oil reservoir with consideration of the second phase of the flow. The essence of this problem from the hydrodynamic point of view involves determining the distribution of reservoir pressure across the reservoir. Pressure distribution in the reservoir, taking into account real changes of fluid and porous medium, in this case is expressed by the following relation

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k(p)}{\mu(p)}\frac{\partial p}{\partial r}\right] = \frac{\partial\left(m\left(1+\beta_m(p-p_0)\right)\right)}{\partial t}, \quad r_q < r < r_k,\ t > 0 \tag{1}$$

here p is reservoir pressure; m is porosity; k is permeability; μ is fluid viscosity; β_m is fluid compression coefficient; r_k is reservoir radius.

As we see, to determine pressure distribution in the reservoir using equation (1), we must know the relationship between permeability and porosity variation vs. pressure.

As we know, the change in porosity of the reservoir with rocks subjected to relaxation deformation, in addition to pressure, is characterized by the following relaxation relationship of the rate of change in porosity over time [1]:

$$m + \tau_m \frac{\partial m}{\partial t} = m_0 \exp(\beta_s(p - p_0)) , \tag{2}$$

here p_0 and m_0 are initial reservoir pressure and porosity, respectively; τ_m is relaxation time of porosity; β_s is the elastic compression coefficient of the rock.

The porosity relaxation time (τ_m) is a time period determining the non-equilibrium change in porosity with respect to the elastic deformation state of the rock (equilibrium state) with decreasing pressure, and characterizing the quantitative difference between the values porosity gets under relaxation deformation of the rock and the values it can get under elastic deformation.

As the relaxation deformation of rocks is mainly characterized by its change according to general law of relaxation in form (2) due to reservoir porosity variation in a smaller interval than reservoir permeability in conditions of decreasing pressure affecting rock porosity, its effect on the reservoir permeability variation from the practical point of view is not taken into account. Since the reservoir permeability variation can be expressed primarily in the process of relaxation variation of the fluid flow rate in the reservoir, depending on the pressure gradient, in the study of non-equilibrium fluid flow problems, as a rule, its variation in this case is described through the concept of flow rate relaxation. On the other hand, since the relaxation variation of the flow rate is expressed as a very small quantity, this quantity may not be taken into account in practical calculations, especially for the flow of a Newtonian fluid in a porous medium. In view of the above, the reservoir permeability variation can be described by the following exponential law [10]:

$$k(p) = k_0 \exp(\alpha_k (p - p_0)) \ , \tag{3}$$

here k_0 is initial absolute permeability of the reservoir; α_k is a coefficient that characterizes the change of permeability depending on pressure.

Thus, to determine the distribution of pressure in the reservoir using equation (1), it is necessary to solve it together with equation (2), taking into account that its permeability is expressed through relationship (3) of its variation depending on pressure.

To determine the reservoir pressure distribution using differential equations (1) and (2), the initial conditions

$$m(r,0) = m_0, \ p(r,0) = p_0 \tag{4}$$

describing the initial state of the reservoir and, respectively, the boundary conditions (5) and (6), corresponding to the case when the well flow rate $q(t)$ is given and the reservoir boundary is impermeable.

$$\frac{2\pi r h k(p)}{\mu(p)} \frac{\partial p}{\partial r}\Big|_{r=r_q} = q(t) \ , \tag{5}$$

$$\frac{\partial p}{\partial r}\Big|_{r=r_k} = 0 \ , \tag{6}$$

here h is the thickness of the reservoir.

3. Solution

We will solve the given problem using the method of averaging the right-hand side of the differential equation.

First, introducing the notation $P = \int_0^p \frac{k(p)}{\mu(p)} dp$ and $\tilde{q} = \frac{q(t)}{2\pi h}$, we express equation (1) and conditions (5) and (6) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P}{\partial r}\right] = \frac{\partial\left(m\left(1+\beta_m(p-p_0)\right)\right)}{\partial t} \ , \tag{7}$$

$$r\frac{\partial P}{\partial r}\Big|_{r=r_q} = \tilde{q} \ , \tag{8}$$

$$\frac{\partial P}{\partial r}\Big|_{r=r_k} = 0 \ . \tag{9}$$

(7) averaging the right-hand side of the equation over the flow area, we write it as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P}{\partial r}\right] = F(t) \ , \tag{10}$$

here

$$F(t)=\frac{2}{r_k^2-r_q^2}\int_{r_q}^{r_k} r\frac{\partial\left(m\left(1+\beta_m(p-p_0)\right)\right)}{\partial t} dr \ . \tag{11}$$

(8) the general solution of the equation looks as follows:

$$P = F(t)\frac{r^2}{4} + C_1(t)\ln r + C_2(t) \tag{12}$$

This expression includes the unknown time-dependent functions $F(t)$, $C_1(t)$ and $C_2(t)$.

Note that for the determination of the unknown quantities $F(t)$, $C_1(t)$ and $C_2(t)$ included in relation (12), in addition to conditions (8) and (9), we will use the condition that the function P in the boundary of the reservoir is equal to the unknown function $P_k(t)$ at each instant of time, i.e.,

$$P(r_k, t) = P_k(t)\ . \tag{13}$$

Considering conditions (8), (9) and (13), the unknown quantities $F(t)$, $C_1(t)$ and $C_2(t)$ are determined as follows:

$$F(t) = -\frac{2\tilde{q}}{r_k^2 - r_q^2},\ C_1(t) = \frac{2r_k^2\tilde{q}}{r_k^2 - r_q^2},\ C_2(t) = P_k(t) + \frac{2r_k^2\tilde{q}}{2(r_k^2 - r_q^2)} - \frac{2r_k^2\tilde{q}}{r_k^2 - r_q^2}\ln r_k\ . \tag{14}$$

Considering these expressions in (12), we obtain the following final expression for determining the function P:

$$P = P_k + \tilde{q}\left[\frac{r_k^2 - r^2}{2(r_k^2 - r_q^2)} + \frac{r_k^2}{r_k^2 - r_q^2}\ln\frac{r}{r_k}\right]. \tag{15}$$

The average value of the function P on the $\boldsymbol{r}$ coordinate is as follows:

$$P_{av} = \frac{2}{r_k^2 - r_q^2}\int_{r_q}^{r_k} rP\,dr\ . \tag{16}$$

From a joint study of (15) and (16), we get:

$$P_{av} = P_k - \tilde{q}\left[\frac{r_k^2 + r_q^2}{4(r_k^2 - r_q^2)} + \frac{r_k^2 r_q^2}{(r_k^2 - r_q^2)^2}\ln\frac{r_q}{r_k}\right]. \tag{17}$$

Writing (15) for $r = r_q$, we obtain:

$$P_q = P_k + \tilde{q}\left[\frac{1}{2} + \frac{r_k^2}{r_k^2 - r_q^2}\ln\frac{r_q}{r_k}\right]. \tag{18}$$

Thus, from (17) and (18), we write:

$$P_{av} - P_q = \tilde{q}\left[\frac{r_k^2}{(r_k^2 - r_q^2)^2}\ln\frac{r_k}{r_q} - \frac{3r_k^2 - r_q^2}{4(r_k^2 - r_q^2)}\right]. \tag{19}$$

On the other hand, applying the rule of approximate calculation of definite integral, we can write the following expressions:

$$P_{av} - P_q = \frac{p_{av} - p_q}{2}\left[\frac{k(p_q)}{\mu(p_q)} + \frac{k(p_{av})}{\mu(p_{av})}\right], \tag{20}$$

$$P_k - P_q = \frac{p_k - p_q}{2}\left[\frac{k(p_q)}{\mu(p_q)} + \frac{k(p_k)}{\mu(p_k)}\right]. \tag{21}$$

Here, p_{av}, p_k are p_q are averaged, pattern and wellbore pressures, respectively.

If we consider these expressions together with expressions (18) and (10), we get:

$$p_q = p_{av} - 2\tilde{q}\,\frac{\frac{r_k^4}{(r_k^2 - r_q^2)^2}\ln\frac{r_k}{r_q} - \frac{3r_k^2 - r_q^2}{4(r_k^2 - r_q^2)}}{\frac{k(p_q)}{\mu(p_q)} + \frac{k(p_{av})}{\mu(p_{av})}}, \tag{22}$$

$$p_q = p_k - 2\tilde{q}\,\frac{\frac{r_k^2}{r_k^2 - r_q^2}\ln\frac{r_k}{r_q} - \frac{1}{2}}{\frac{k(p_q)}{\mu(p_q)} + \frac{k(p_k)}{\mu(p_k)}}. \tag{23}$$

It should be noted that when examining relevant hydro-gas dynamics problems in the oil and gas production practice, the determination of the pattern and well bottom pressure variation over time is carried out by taking the values of the pattern pressure as approximately equal to the variation of the averaged reservoir pressure ($p_k \approx \boldsymbol{p_{av}}$) [11].

In order to accept the approximate equality $p_k \approx p_{av}$ in relations (22) and (23), the copies of fractions on the right-hand side should be close to each other. Let us check how valid this assumption is. Given that the relative error of the boundary located on the right-hand side of the second relation with respect to the corresponding boundary of the first relationship is $r_q \ll r_k$, it is determined as follows:

$$100 \cdot \left| \frac{\frac{r_k^4}{\left(r_k^2-r_q^2\right)^2}\ln\frac{r_k}{r_q} - \frac{3r_k^2-r_q^2}{4\left(r_k^2-r_q^2\right)} - \frac{r_k^2}{r_k^2-r_q^2}\ln\frac{r_k}{r_q} + \frac{1}{2}}{\frac{r_k^2}{r_k^2-r_q^2}\ln\frac{r_k}{r_q} - \frac{1}{2}} \right| \approx 100 \cdot \left| \frac{1}{4\ln\frac{r_k}{r_q} - 2} \right|.$$

Calculations show that when $r_q = 0.1$ m , $r_k = 100$ m , 200 m , 500 m and 1000 m, the values of this error are 3.9%, 3.52%, 3.11%, 2.87%, respectively. That is, as the radius of the well's zone of effect (the distance from the bottom of the well to the boundary) increases, the approximate equation $p_k \approx p_{av}$ can be taken with a smaller error. An analysis shows that in such cases, the wellbore pressure can be calculated with high practical accuracy using relation (22) or (23).

In general, if the variation of the averaged reservoir pressure is known, the variation of the wellbore pressure can be determined with the help of equation (22), and the variation of the pattern pressure over time with the help of equation (23) after the variation of the wellbore pressure is determined.

The variation of the averaged reservoir pressure over time can be carried out according to the rule given in [5], based on the following considerations.

Based on (11) and (14), we can write the following equality:

$$\frac{2}{r_k^2-r_q^2}\int_{r_q}^{r_k} r\frac{\partial\left(m\left(1+\beta_m(p-p_0)\right)\right)}{\partial t}dr = -\frac{2\tilde{q}}{r_k^2-r_q^2}\,.$$

Performing some simple manipulations in this equality and if we integrate over time over the interval $(0, t)$, we get:

$$\tilde{q} = -\int_{r_q}^{r_k} r\frac{\partial\left(m\left(1+\beta_m(p-p_0)\right)\right)}{\partial t}dr\,.$$

If we integrate both sides of equality (18) in the fragment $[0;t]$ and consider the form

$$m(0) = m_0,\ p(0) = p_0 \tag{24}$$

corresponding to the dependence of initial conditions (4) of porosity and formation pressure on one variable, we get:

$$\tilde{Q} = \int_0^t \tilde{q}\,dt = -\int_{r_q}^{r_k} r\left(m\left(1+\beta_m(p-p_0)\right) - m_0\right)dr\,. \tag{25}$$

For reservoir-averaged estimates of the quantity $m\left(1+\beta_m(p-p_0)\right)$, the following equation is true:

$$m_{av}\left(1+\beta_m(p_{av}-p_0)\right) = \frac{2}{r_k^2-r_q^2}\int_{r_q}^{r_k} r\left(m\left(1+\beta_m(p-p_0)\right)\right)dr\,. \tag{26}$$

Here m_{av} is averaged values of reservoir porosity across the reservoir.

Considering (26) in (25) and returning to the substitution $\tilde{q} = \frac{q(t)}{2\pi h}$, we get:

$$Q = \pi\left(r_k^2 - r_q^2\right)h\left(m_0 - m_{av}\left(1+\beta_m(p_{av}-p_0)\right)\right). \tag{27}$$

Here, $Q = \int_0^t q\,dt$.

Relation (27) expresses the material balance equation of the single-phase fluid flow in the reservoir, written in integral form.

Using the form of equation (27) written according to (2) for the averaged values of reservoir porosity by r (in this case, the symbol ∂ is replaced by the symbol D), we obtain the following analytical and differential relations for determining the averaged reservoir pressure (the quantity p_{av}) by the values of the quantity m_{av} determined from the joint study of the resulting equation:

$$p_{av} = p_0 + \left(\left(m_0 - \frac{Q}{\pi(r_k^2 - r_q^2)h}\right)\frac{1}{\beta_m m_{av}} - \frac{1}{\beta_m}\right), \quad (28)$$

$$\frac{dm_{av}}{dt} = \frac{m_0}{\tau_m}\exp\left(\left(m_0 - \frac{Q}{\pi(r_k^2 - r_q^2)h}\right)\frac{\beta_s}{\beta_m m_{av}} - \frac{\beta_s}{\beta_m}\right) - \frac{m_{av}}{\tau_m}. \quad (29)$$

Equation (29) is solved taking into account the initial condition $m(0) = m_0$.

Thus, after determining the quantity p_{av} with the help of (27) and (28), the variation of the wellbore and pattern pressures over time, with the flow rate of the well given, is determined with the help of equations (22) and (23).

The reservoir pressure distribution is determined using solution (15) and methods of mathematical solution of the relation

$$\int_{p_k}^{p} \frac{k(p)}{\mu(p)} dp = \frac{q}{2\pi h}\left(\frac{r_k^2 - r^2}{2(r_k^2 - r_q^2)} + \frac{r_k^2}{r_k^2 - r_q^2}\ln\frac{r}{r_k}\right). \quad (30)$$

obtained by returning to the originally adopted notations of the function P and the quantity $\tilde{q}$, respectively. In the special case, when the permeability of the reservoir and the fluid viscosity are assumed to be constant (k_0 and μ_0, respectively), we obtain obviously calculable relation (31) for determining the pressure distribution across the reservoir from the last expression:

$$p = p_k + \frac{q\mu_0}{2\pi k_0 h}\left(\frac{r_k^2 - r^2}{2(r_k^2 - r_q^2)} + \frac{r_k^2}{r_k^2 - r_q^2}\ln\frac{r}{r_k}\right). \quad (31)$$

4. Conclusion

The problem of determining the indicators of development of oil reservoirs whose rocks are subjected to relaxation deformation in single-phase fluid flow is formulated, and its approximate solution is obtained using the averaging method and the approach based on the use of material balance equations. The novelty of the result of the obtained solution consists in the fact that for a given well flow rate, determination of unknown reservoir development indicators can be carried out using mathematical relations, which can be relatively simply implemented computationally and have practical accuracy acceptable for use in relevant practical calculations.

References

[1] А.М. Кулиев, Б.З. Казымов, Деформация горных пород и ее влияние на их фильтрационно-емкостные свойства и на процессы фильтрации и разработки месторождений нефти и газа, Баку, Элм, (2009) 88 p. [In Russian: A.M. Guliyev, B.Z. Kazimov, Deformation of rocks and its influence on their filtration-capacitative properties and on the processes of filtration and development of oil and gas fields, Baku, Elm].

[2] Г.И. Джалалов, Т.М. Ибрагимов, А.А. Алиев, Е.В. Горшкова, Моделирование и исследование фильтрационных процессов глубокозалегающих месторождений нефти и газа, Баку, “Elm və Təhsil” ИПП, (2018) 384 p. [In Russian: G.I. Jalalov, T.M. Ibrahimov, A.A. Aliyev, Ye.V. Gorshkova, Modeling and study of filtration processes of deep oil and gas fields, Baku, Elm və Təhsil IPP].

[3] Ю.П. Желтов, Разработка нефтяных месторождений, Москва, Недра, (1986) 332 p. [In Russian: Yu.P. Zheltov, Development of oil fields, Moscow, Nedra].

[4] Ю.М. Молокович, П.П. Осипов, Основы теории релаксационной фильтрации, Казань, Казанский Государственный Университет, (1987) 118 p. [In Russian: Yu.M. Molokovich, P.P. Osipov, Fundamentals of the theory of relaxation filtration, Kazan, Kazan State University].

[5] B.Z. Kazımov, K.K. Nəsirova, Relaksasiyalı mühitə malik qaz yatağının işlənmə göstəricilərinin təqribi təyini üsulu, "Gənc alimlərin əsərləri". No.2 (2009) pp.81-92. [In Azerbaijani: B.Z. Kazimov, K.K. Nasirova, A method of approximate determination of indicators of development of a gas field with a relaxation media].

[6] Э.В. Маммадов, Б.З. Казымов, Методика определения оптимального режима работы газоконденсатной скважины с учетом деформации горных пород пласт-коллектора, Международный научный журнал "Ученый XXI века". 65 No.6-3 (2020) pp.3-7 [In Russian: E.V. Mammadov, B.Z. Kazimov, A technique for determining the optimal mode of operation of a gas condensate well, taking into account the deformation of reservoir rocks, International Scientific Journal "Scientist of the XXI century".].

[7] Б.З. Казымов, Р.М. Зейналов, Выбор оптимального режима эксплуатации газовой скважины с учетом ползучей деформации горных пород, Материалы III научно-практической конференции "Современное программирование" (Нижневартовск, 27-29 ноября 2020 года), Нижневартовский государственный университет. (2021) pp.159-162. [In Russian: B.Z. Kazimov, R.M. Zeynalov, Choice of optimal mode of operation of gas well taking into account creeping deformation of rocks, Proceedings of III Scientific-Practical Conference "Modern Programming" (Nizhnevartovsk, November 27-29, 2020), Nizhnevartovsk State University].

[8] Б.З. Казымов, Т.А. Самедов, С.Г. Новрузова, Э.В. Гадашева, Определение коллекторских свойств газовых пластов в условиях ползучести горных пород с ядром Абеля, Технологии нефти и газа, 133 No.2 (2021) pp.31-33 [In Russian: B.Z. Kazimov, T.A. Samedov, S.G. Novruzova, E.V. Gadasheva, Determination of reservoir properties of gas reservoirs in conditions of rock creep with Abel core, Tekhnologii neft i gaza].

[9] Б.З. Казымов, Р.М. Зейналов, Определение динамики потребного количества скважин при разработке глубокозалегающих нефтяных залежей в режиме растворенного газа, Материалы IV Международной научно-практической конференции (Россия, г. Нижневартовск, 08 декабря 2021 г.) на тему "Современное программирование", Нижневартовск. (2022) pp.133-137. [In Russian: B.Z. Kazimov, R.M. Zeynalov, Determination of the dynamics of the required number of wells in the development of deep oil reservoirs in the dissolved gas mode, Proceedings of the IV International Scientific Conference "Modern Programming" (Russia, Nizhnevartovsk, December 08, 2021), Nizhnevartovsk].

[10] К.С. Басниев, Н.М. Дмитриев, Г.Д. Розенберг, Нефтегазовая гидромеханика: учебное пособие для вузов, Москва-Ижевск, Институт компьютерных исследований, (2005) 544 p. [In Russian: K.S. Basniyev, N.M. Dmitriyev, G.D. Rosenberg, Oil and gas fluid mechanics: textbook for universities, Moscow-Izhevsk, Institute for Computer Research].

[11] С.Н. Закиров, Теория и проектирование разработки газовых и газоконденсатных месторождений, Москва, Недра, (1989) 334 p. [In Russian: S.N. Zakirov, Theory and engineering of gas and gas condensate field development, Moscow, Nedra].