Informatics and Control Problems 42 Issue 2 (2022)
journal homepage: www.icp.az

Accuracy comparison of signal recognition methods on the example of a family of successively horizontally displaced curves

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ARTICLE INFO

Article history:
Received 11.09.2022
Received in revised form 22.09.2022
Accepted 09.10.2022
Available online 18.11.2022

Keywords:
Signal
Recognition method
Accuracy
Comparative assessment

ABSTRACT

In some cases, to compare recognition methods the criterion of the total percentage ratio of the proximity of recognized signals to the reference ones is applied. This study proposes a slightly different approach, involving a numerical evaluation when comparing two or more signal recognition methods on the example of an artificially created family of successively and uniformly horizontally displaced curves.

1. Introduction

We consider a new approach to a comparative assessment of the adequacy of signal recognition methods: the algorithms of Dynamic Time Warping (DTW) [1] and Derivative Dynamic Time Warping (DDTW) [2]. It is known that the Euclidean distance is used mainly to determine the proximity between signals. Typically, to assess the adequacy of the recognition method, a certain number of signals is selected, for which recognition is carried out, and by the number of recognized signals the degrees of adequacy (accuracy) of the methods used is established, and these methods are subsequently used to compare the recognition methods themselves on the following principle: the higher the accuracy, the better the recognition method [1, 2, 3]. Nevertheless, this approach has its disadvantages: in addition to high computational costs, it implies the presence of uncertainty and elements of randomness.

In [4], the so-called Itakura parallelogram is used as part of the DTW algorithm. The results obtained by the modified Itakura algorithm are compared with the corresponding results obtained on the basis of DTW. 85 databases are used for comparison here. For this purpose, it would be very difficult to apply known sequences of functions converging to a given one, such as the Fourier series

$$f = \sum_{-\infty}^{\infty} c_k e_k$$

where $e_k = cos(kt) + sin(kt)$ $(k\epsilon Z)$ which converges to the function f in the sense of the metric L_0^2. In addition to the above, it should be noted that the pointwise convergence for a sufficient regular function $f\epsilon L_0^2$ proved by Carleson's theorem [5, pp. 46-47], also has important practical applications.

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www.icp.az/2022/2-10.pdf https://doi.org/10.54381/icp.2022.2.10
2664-2085/

In this article we propose using an artificially created family of successively and uniformly horizontally displaced curves to compare recognition methods.

2. Problem statement

The main aim of the study is to create an algorithm for a comparative quantitative assessment of signal recognition methods. To achieve this aim, a basic signal (reference) and its horizontally uniformly and successively displaced analogs are selected. On the basis of an artificially created family of such curves it is required to carry out DTW and DDTW signal recognition using methods, as well as a comparative assessment of the accuracy of these methods on the basis of the numerical criterion of proximity.

3. Comparison of DTW and DDTW recognition methods

Let us consider some features of the DTW and DDTW algorithms. In particular, the difference between the DTW algorithm and the trivial method of recognition based on the application of the Euclidean distance is shown in Fig. 1.

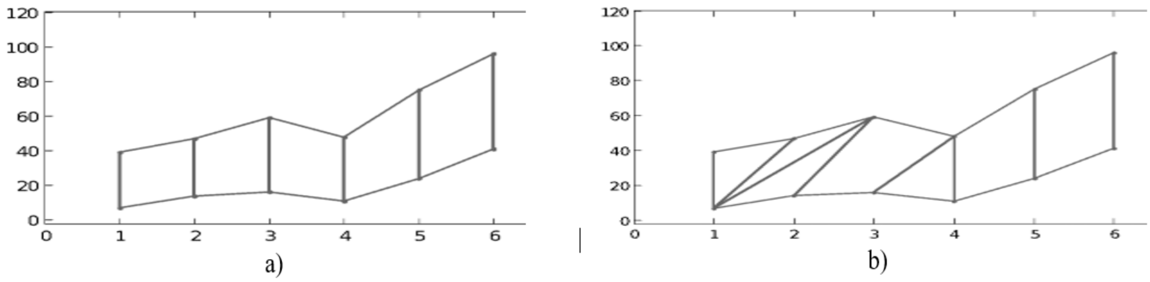


Fig. 1. a) Euclidean distance; b) DTW

The DTW algorithm finds the "best" time transformation between two signals, establishes paths and calculates distances between signals along these paths. At the same time, the trivial way of estimating the proximity of signals, based on the application of the Euclidean distance between two signals, implies directly the use of relevant samples.

Let us first briefly describe the DTW algorithm for clarity and perform the calculations using one example. Suppose we have two numerical sequences $\{f_1, f_2, ..., f_n\}$ and $\{g_1, g_2, ..., g_m\}$ (see Fig. 2) with lengths *n* and *m*, respectively. The algorithm starts with the calculation of the local deviations between the elements of the two sequences using different types of deviations. The most common method for calculating deviations is the method that calculates the absolute deviation between the values of two elements (Euclidean distance). The result is a quadratic deviation matrix consisting of *n* rows and *m* columns:

$$d_{ij} = \left(f_i - g_j\right)^2, i = 1, \dots, n, j = 1, \dots, m. \tag{1}$$

The minimum distance in the matrix between sequences is determined using the following criterion:

$$\begin{cases} DTW(f_i, g_j)^2 = d_{ij} + min\left\{DTW(f_i, g_{(j-1)})^2, DTW(f_{(i-1)}, g_j)^2, DTW(f_{(i-1)}, g_{(j-1)})^2\right\}, \\ DTW(f_1, g_1)^2 = d_{11}. \end{cases} \tag{2}$$

where $DTW(f_n, g_m)^2$ is the minimal (squared) distance between the sequences {f_1 , f_2 , f_n} and {g_1 , g_2 , ..., g_m}.

The following three conditions are satisfied for DTW:

1. Monotonicity – both indices (*i* and *j*) are consistently increasing.
2. Continuity – in one step the indices (*i* and *j*) increase by no more than one.
3. The sequence of building "paths" starts in the lower left corner and ends in the upper right corner.

The DTW algorithm is applied with a "limit" and "with no limit" on the size of the so-called "window". The size of the window w determines the number of allowed sample, allowing for right and left comparisons. The total number of samples is $2w + 1$, so that, for instance, the comparison of the *i*-th sample of one signal f_i with the *j*-th sample of another signal g_j must satisfy the inequality $|i - j| \leq w$.

Using Table 1 as an example, consider the following trivial case of recognition with no limit on window size.

Table 1.

Signal *f* and *g*

Sample number	1	2	3	4	5	6
Values of f_i (i=1÷6)	39	47	59	48	75	96
Values of g_j (j=1÷6)	7	14	16	11	24	41

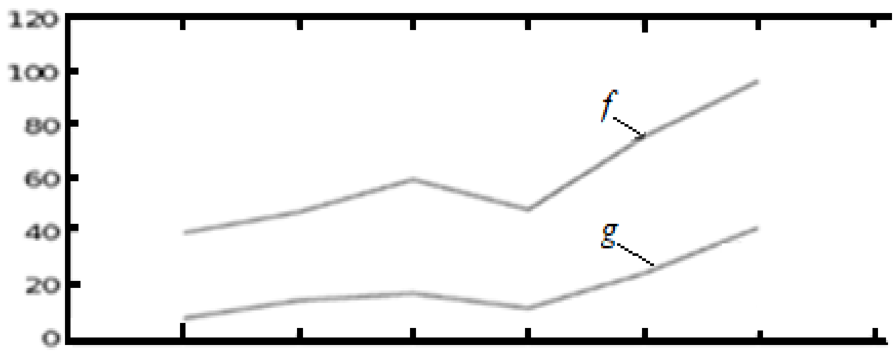


Fig. 2. Recognized signals *f* and *g*

In the Euclidean metric, the signals are compared as shown in Fig. 3. In this case, if the squared Euclidean distance between the corresponding values of the two samples is applied, the absolute deviation is calculated from formula (1). For instance, for the first samples $d_{11}=(f_1-g_1)^2 = (39-7)^2 = =1024$, and for the remaining samples, the results can be written as a two-dimensional deviation matrix, presented as Table 2.

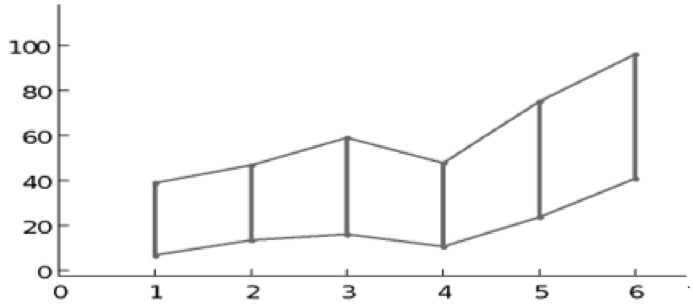


Fig. 3. Comparison of signals using the Euclidean distance

Table 2.

Deviation matrix

f_6	7921	6724	6400	7225	5184	3025
f_5	4624	3721	3481	4096	2601	1156
f_4	1681	1156	1024	1369	576	49
f_3	2704	2025	1849	2304	1225	324
f_2	1600	1089	961	1296	529	36
f_1	1024	625	529	784	225	4
	g_1	g_2	g_3	g_4	g_5	g_6

Now we have to find the "best" path that indicates which samples we will compare. For instance, if we compare only the corresponding samples, this path corresponds to a diagonal in the deviation matrix (shown in bold).

Table 3.

Corresponding signal samples

f_6	7921	6724	6400	7225	5184	**3025**
f_5	4624	3721	3481	4096	**2601**	1156
f_4	1681	1156	1024	**1369**	576	49
f_3	2704	2025	**1849**	2304	1225	324
f_2	1600	**1089**	961	1296	529	36
f_1	**1024**	625	529	784	225	4
	g_1	g_2	g_3	g_4	g_5	g_6

In this case the distance is **10957.**

Table 4.

An example of a different path

f_6	7921	6724	6400	7225	5184	**3025**
f_5	4624	3721	3481	4096	**2601**	1156
f_4	1681	1156	**1024**	**1369**	576	49
f_3	**2704**	**2025**	1849	2304	1225	324
f_2	**1600**	1089	961	1296	529	36
f_1	**1024**	625	529	784	225	4
	g_1	g_2	g_3	g_4	g_5	g_6

1024+1600+2704+ 2025+1024+1369+2601+3025=15372

This is not the "best" path, because the value **15372 is greater than 10957**.

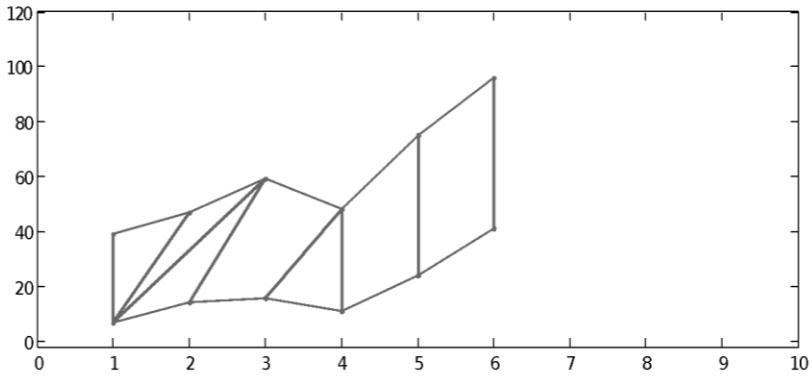


Fig. 4. Comparison corresponding to Table 4

To find the "best" way using the deviation matrix, we build the DTW matrix by formula (2). Let us make several calculations by formula (2):

$$DTW(f_1, g_1)^2 = d_{1,1} = 1024,$$

$$DTW(f_2, g_1)^2 = d_{21} + min\{DTW(f_2, g_0)^2, DTW(f_1, g_1)^2, DTW(f_1, g_0)^2\} =$$

$$=1600 + min\{DTW(f_2, g_0)^2, 1024, DTW(f_1, g_0)^2\} = 1600 + 1024 = 2624,$$

Since in this case $DTW(f_2, g_0)^2, DTW(f_1, g_0)^2$ are not determined, they are not taken into account in finding $min\{DTW(f_2, g_0)^2, 1024, DTW(f_1, g_0)^2\}$ are not taken into account (because the zero indices are not determined).

$$DTW(f_1, g_2)^2 = d_{12} + min\{DTW(f_1, g_1)^2, DTW(f_0, g_2)^2, DTW(f_0, g_1)^2\} =$$

$$=625 + min\{1024, DTW(f_2, g_0)^2, DTW(f_1, g_0)^2\} = 625 + 1024 = 1649,$$

$$DTW(f_2, g_2)^2 = d_{22} + min\{DTW(f_2, g_1)^2, DTW(f_1, g_2)^2, DTW(f_1, g_1)^2\} =$$

$$=1089 + min\{2624, 1649, 1024\} = 1089 + 1024 = 2113$$

Table 5.

DTW matrix

6	19554	15739	14867	15692	13060	7777
5	11633	9015	8467	9082	7876	4752
4	7009	5294	4986	5331	5275	3596
3	5328	4138	3962	4914	4699	3547
2	2624	2113	2610	3474	3491	3223
1	1024	1649	2178	2962	3187	3191
DTW	1	2	3	4	5	6

After the DTW matrix is built, it is necessary to start from the top right corner and come to the bottom left corner. When doing this, we begin from the top right corner and to move either diagonally or forward to the right or down to the left and to choose from three possible directions where DTW value is minimal. Then we get:

Table 6.

The "best" path in the DTW matrix

6	19554	15739	14867	15692	13060	**7777**
5	11633	9015	8467	9082	7876	**4752**
4	7009	5294	4986	5331	5275	**3596**
3	5328	4138	3962	4914	4699	**3547**
2	2624	2113	2610	3474	3491	**3223**
1	**1024**	**1649**	**2178**	**2962**	**3187**	3191
DTW	1	2	3	4	5	6

Now we have a path (Table 7). Let us indicate this path on the deviation matrix and find for this path the Euclidean distance (squared) between the two signals (the new value for the Euclidean distance will be the "best"):

Table 7.

The "best" path in the deviation matrix

f_6	7921	6724	6400	7225	5184	**3025**
f_5	4624	3721	3481	4096	2601	**1156**
f_4	1681	1156	1024	1369	576	**49**
f_3	2704	2025	1849	2304	1225	**324**
f_2	1600	1089	961	1296	529	**36**
f_1	**1024**	**625**	**529**	**784**	**225**	4
	g_1	g_2	g_3	g_4	g_5	g_6

For this path the Euclidean distance is equal to:

1024+625+529+784+225+36+324+49+1156+3025=7777.

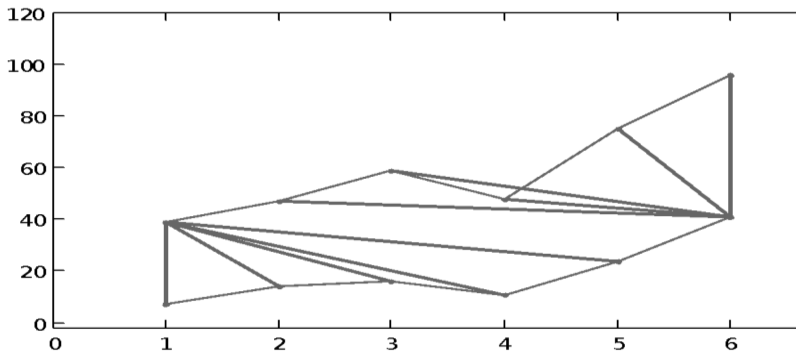


Fig. 5. Corresponding "best" comparison paths

In the case where the window size is $w = 2$, the calculated values for the deviation and DTW matrices are shown in Tables 8 and 9.

Table 8.

Deviation matrix for the window size $w = 2$

f_6				7225	5184	3025
f_5			3481	4096	2601	1156
f_4		1156	1024	1369	576	49
f_3	2704	2025	1849	2304	1225	
f_2	1600	1089	961	1296		
f_1	1024	625	529			
	g_1	g_2	g_3	g_4	g_5	g_6

Table 9.

DTW matrix for the window size $w = 2$

6				15692	13060	**8929**
5			8467	9082	7876	**5904**
4		5294	4986	5331	5275	**4748**
3	5328	4138	3962	4914	**4699**	
2	2624	2113	2610	**3474**		
1	**1024**	**1649**	**2178**			
DTW	1	2	3	4	5	6

The path is shown in the following table:

Table 10.

Path for the window size $w = 2$

Sample number of signal f	6	5	4	3	2	1	1	1
Sample number of signal g	6	6	6	5	4	3	2	1

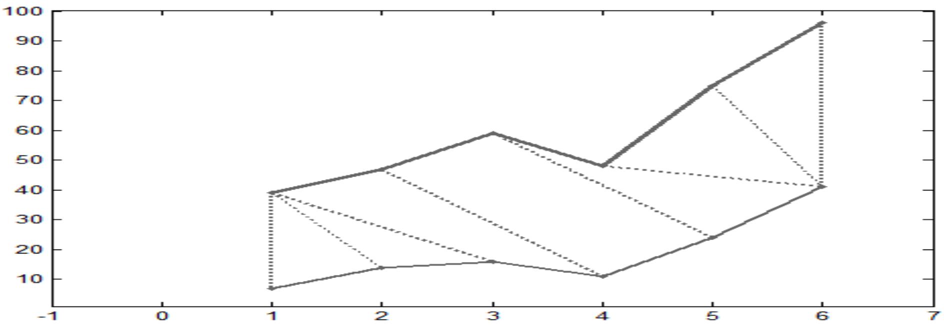


Fig. 6. Comparison corresponding to Table 10

In order to solve the problem we built successive "artificial" signals according to the following principle: a signal is selected and by means of horizontal shifts successive signals are formed. In particular, as a standard conditionally chosen signal s and 6 of its mappings shifted to the right by 5 samples: s_1, s_2, ..., s_6 (see Fig. 7).

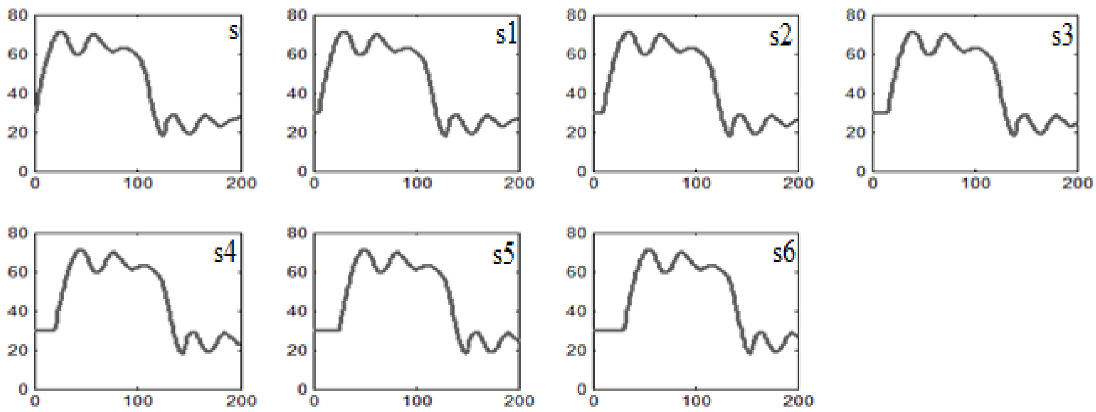


Fig. 7. Signal s and its 6 analogs shifted to the right by 5 samples: s_1 , s_2 , ..., s_6

Thus, we have obtained 7 signals: the reference and its 6 analogs shifted to the right $s, s_1, s_2, .., s_6$. To recognize the signals $s, s_1, s_2, ..., s_6$ we applied the DTW, Euclidean, and DDTW algorithms. However, it should be noted that, unlike DTW, the DDTW algorithm uses the derivatives at the corresponding points instead of the numerical values of the samples. The calculated results of the distances between the signals $s, s_1, s_2, ..., s_6$ for DTW and Euclidean algorithms are summarized in Tables 1 and 2.

Table 11.

Distance matrix by the DTW algorithm

	s	s_1	s_2	s_3	s_4	s_5	s_6
s	0	1.73	45.76	99.66	143.41	175.74	203.61
s_1	1.73	0	1.73	45.73	99.59	143.15	175.50
s_2	45.76	1.73	0	2	45.72	99.44	142.97
s_3	99.66	45.73	2	0	1.73	45.64	99.28
s_4	143.41	99.59	45.72	1.73	0	1.41	45.50
s_5	175.74	143.15	99.44	45.64	1.41	0	1.73
s_6	203.61	175.50	142.97	99.28	45.50	1.73	0

Table 12.

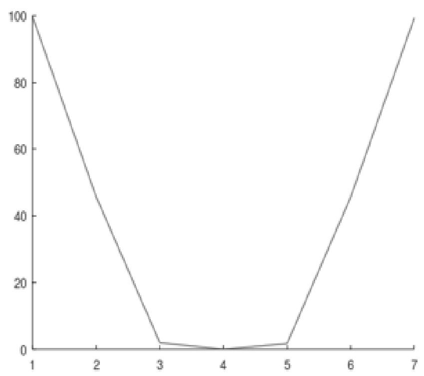
Distance matrix by the Euclidean algorithm

	s	s_1	s_2	s_3	s_4	s_5	s_6
s	0	59.79	112.01	156.04	190.96	217.58	238.60
s_1	59.79	0	59.75	111.95	155.92	190.77	217.38
s_2	112.01	59.75	0	59.73	111.88	155.79	190.63
s_3	156.04	111.95	59.733	0	59.67	111.76	155.69
s_4	190.96	155.92	111.88	59.67	0	59.61	111.71
s_5	217.58	190.77	155.79	111.76	59.61	0	59.59
s_6	238.6	217.38	190.63	155.69	111.71	59.59	0

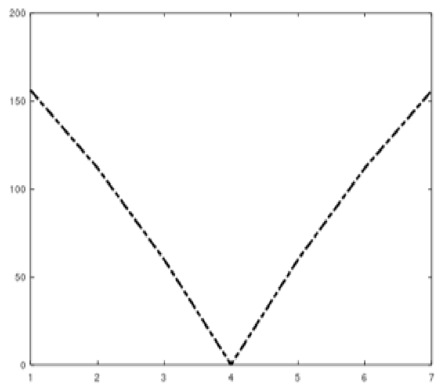
According to [6], intuition and practical experience (heuristic knowledge) of the researcher are the decisive factors in recognition. Based on these considerations, on the basis of empirical analysis we propose three criteria, which are formulated as follows:

- criterion 1 (*method stability*): with the distance of the signals from the reference, the distances between them should increase, rather than vary step-wise;
- criterion 2 (*method sensitivity*): for a particular signal the distances from the left and right standing signals must be almost equal. In case the left and right standing signals are completely symmetrical with the given signal, the values are equal. To verify this criterion numerically, for instance (in case these distances are non-zero) for some signal the distance from the standing left signal can be divided by the distance from the standing right signal, thus we should get a number close to one;
- criterion 3 (*method speed*): as the signals get closer, the speed of convergence of distance values increases. Here, the speed of convergence of distance values means the difference between the current and the next distance values divided by the current distance value.

Further, on the basis of these criteria, first we compare the results of recognition using the DTW and Euclid methods. A check by the first criterion shows that **criterion 1** for all signals is not violated. For example, the obtained numerical results for s_3 are visually shown in Fig. 8:



a)



b)

Fig. 8. Graphs of the variation of the distances from s_3 to the remaining signals by the DTW method (a) and by the Euclidean algorithm (b)

Comparison according to criterion 2 using the DTW algorithm shows that the maximum value takes place for the fifth and sixth signals and is **1.2248:**

$$\mathbf{1.7321/1.4142 = 1.2248}$$

Table 13.

Comparison by criterion 2 according to the DTW algorithm

	s	s_1	s_2	s_3	s_4	s_5	s_6
s_5	143.41	99.599	45.727	**1.732**	0	**1.414**	45.508
s_6	175.74	143.150	99.443	45.640	**1.414**	0	**1.732**

Comparison by criterion 2 according to the Euclidean algorithm shows that the maximum value takes place for s_3 and s_4 and is 0.99902:

$$59.674/59.73 \approx 0.99902, 59.59/59.61 \approx 0.99902.$$

Table 14.

Comparison by criterion 2 using the Euclidean algorithm

	s	s_1	s_2	s_3	s_4	s_5	s_6
s_4	156.04	111.95	**59.73**	0	**59.67**	111.76	155.69
s_6	217.58	190.77	155.79	111.76	**59.61**	0	**59.59**

According to the results of comparisons by criterion 2, the Euclidean algorithm outperforms the DTW algorithm numerically by 1.2248/0.99902 ≈1.2260 times.

The numerical results for criterion 2 are shown in Fig. 9:

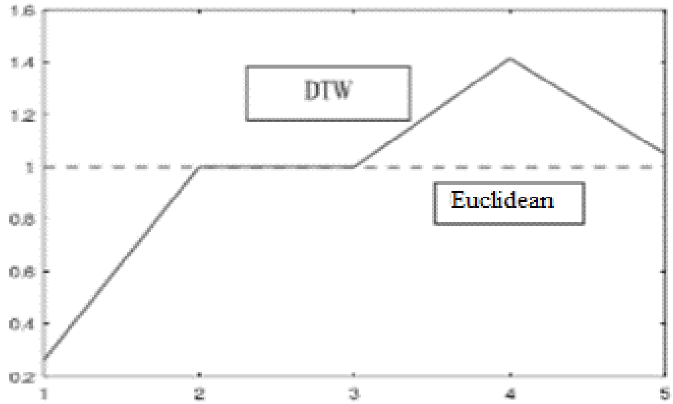


Fig. 9. Graphs of the deviation of the values calculated by the DTW method and by the Euclid method from one

The comparison by criterion 3 for ***s*** using the DTW and Euclidean algorithms is shown in Table 15.

Table 15.

Comparison by criterion 3

DTW	0.1368	0.1839	0.3050	0.5408	0.9621	1
Euclidean	0.0881	0.1223	0.1828	0.2821	0.4661	1

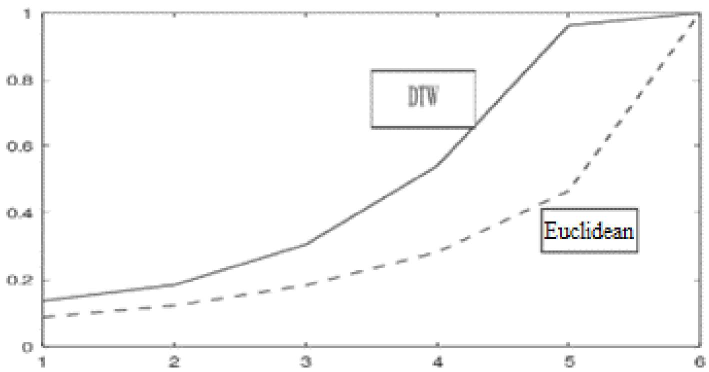


Fig. 10. Graphs of **the speed of** DTW and Euclidean **methods**

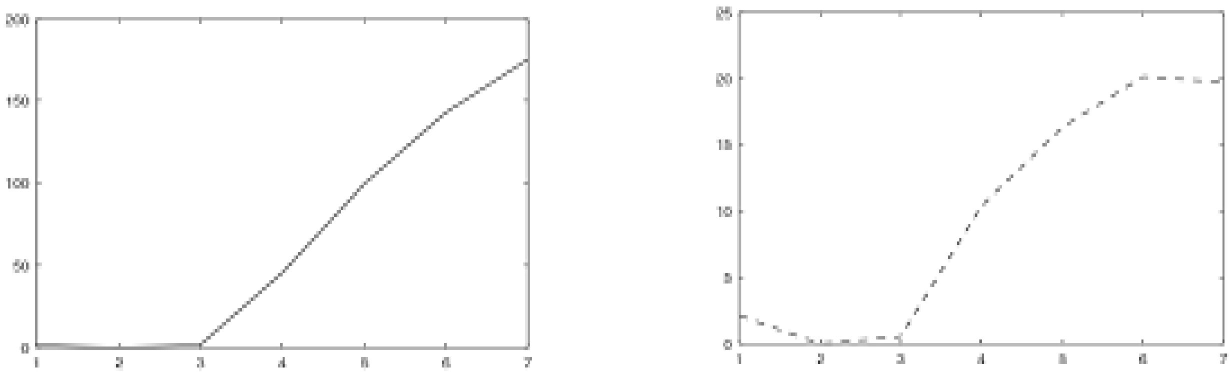
Here are the results of distance calculations using the DDTW algorithm:

Table 16.

Distance matrix by the DDTW algorithm

	s	s_1	s_2	s_3	s_4	s_5	s_6
s	0	2.136	10.671	16.475	20.281	19.906	18.995
s_1	2.136	0	0.559	10.398	16.3	20.145	19.741
s_2	10.671	0.559	0	0.559	10.398	16.304	20.106
s_3	16.475	10.398	0.559	0	0.559	10.413	16.263
s_4	20.281	16.3	10.398	0.559	0	0.790	10.374
s_5	19.906	20.145	16.304	10.413	0.79057	0	0.829
s_6	18.995	19.741	20.106	16.263	10.374	0.82916	0

For instance, for s_1 the obtained numerical results are shown in Fig. 3:



a) **b)**

Fig.11. Demonstration of the variation of the distances from s_1 to other signals using: a) DTW; b) DDTW

For DDTW, the maximum value for criterion 2 takes place for the second signal and is $2.136/0.55902 \approx 3.821$. The DTW algorithm outperforms the DDTW algorithm numerically by $3.821/1.2248 \approx 3.119$ times.

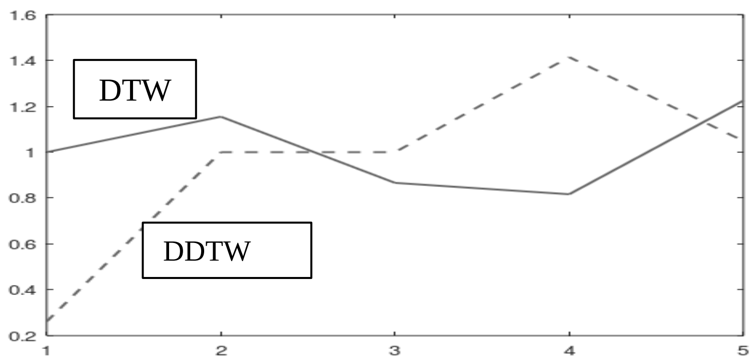


Fig. 12: Graphs of the deviations of the values calculated by DTW and DDTW methods from one

Table 17.

Comparison by criterion 3 for s_2 using DTW and DDTW algorithms

DTW	0.1368	0.1839	0.3050	0.5408	0.9621	1
DDTW	-0.0479	-0.0188	0.1876	0.3522	0.7998	1

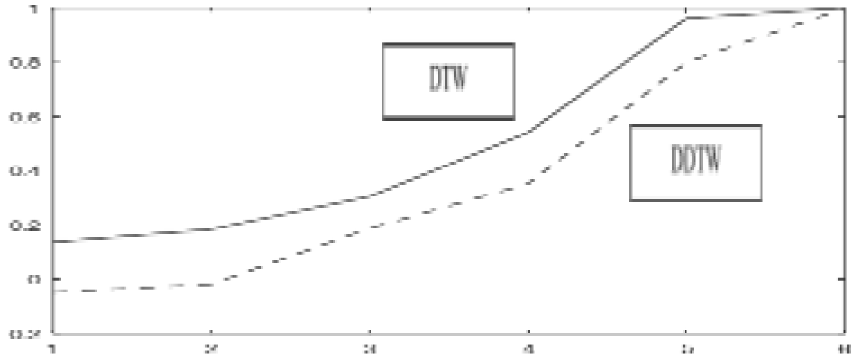


Fig. 13. Graphs of the speeds of DTW and DDTW methods

4. Conclusion

The calculations have shown the validity of the DTW method by three criteria. By criterion 2 (speed) DTW surpassed the Euclidean method.

The calculations also showed the validity of the DDTW method by two criteria (except for the first criterion). Nevertheless, the accuracy of the DTW method by criteria 2 and 3 still surpassed the accuracy of the DDTW method. Thus, our proposed method of artificial successive signals is suitable both for the analysis of an individual method, and for a comparative assessment of the adequacy of various recognition methods.

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