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Technologies for analyzing the condition of transport system facilities based on noise characteristics

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ABSTRACT

The article considers the causes of malfunctions, damage and failures of transport system facilities. It is noted that traditionally monitoring systems are created to control the technical condition of transport system facilities. However, the algorithms and software of the existing systems do not take into account the fact that the occurrence of damage to transport system facilities is accompanied by the emergence of additive noises, which are superimposed on the useful signals coming from the corresponding sensors and correlated with them. Technologies are developed for analyzing the technical condition of transport system facilities as a result of calculating the relay and normalized cross-correlation functions, as well as the correlation coefficient between the useful signal and the noise of the noisy signal. Algorithms for analyzing the rate of change in the technical condition of transport system facilities by constructing a confidence interval for the mathematical expectation of the noise are proposed. Technologies for analyzing the performance of transport system facilities based on the proposed characteristics of the noise are developed. The results of a computational experiment are given and a comparative analysis is conducted. It is noted that the application of the developed algorithms and technologies allows detecting the latent period of malfunction occurrence, reduces the risk of destruction of transport system facilities as a result of timely routine and overhaul repairs.

1. Introduction

It is known that the transport system consists of transport infrastructure, transport companies, transport vehicles and management. The transport system serves the transportation needs of human beings and includes means of transportation, transportation facilities, and the environment. Transport infrastructure includes transport networks or communication routes, i.e., motor roads, railroads, air corridors, canals, pipelines, bridges, tunnels, waterways, etc., as well as transportation hubs, which include airports, railway stations, bus stops, and ports. Along with education, health, energy, communications, etc., the transportation system plays an important role in achieving social, economic, foreign policy and other government priorities. Today, the transport system occupies a special place in the development of every country's economy. Creating a transport system requires massive capital investments, construction, reorganization and reconstruction of a colossal number of transport hubs. In this regard, in modern conditions there is a need to ensure the safety of the

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functioning of the transport system, adequate monitoring and timely detection and elimination of malfunctions in the latent period of their occurrence, carrying out prophylactic work to prevent accidents and destruction [1-3].

To improve the safety of transport system facilities, monitoring systems based on modern information technologies are created and applied. Monitoring and warning systems track the condition of roads, railroads, air corridors, canals, pipelines, bridges, tunnels, waterways, airports, railway stations, bus stops, ports, etc. Data on the condition of transport system facilities are transmitted in the form of signals to the central monitoring station. The received digital information is logged, processed, analyzed and conclusions are drawn about the condition of the transport system as a whole, as well as the presence of defects and the degree of damage in each facility. In addition, conclusions are made about the causes that led to this or that malfunction [1-4].

However, the existing monitoring systems do not provide control of the early latent period of change in the facilities of the transport system. In [4-10] it is shown that even smallest damage to transport system facilities is accompanied by the appearance of additive noises, which are correlated with the useful signals of noisy signals coming from the corresponding sensors. At the same time, the dynamic of damage development is reflected in the noise characteristics, which change over time depending on the degree of malfunction and the rate of its development. One of these characteristics is the value of the confidence interval for the mathematical expectation of the noise.

Studies have shown that the use of technologies to calculate the degree of correlation of the useful signal and the noise, as well as the interval of variation of the mathematical expectation of noise, allow to identify the early latent period of defect and damage formation, determine the dynamics of its development, as well as reduce the accident risks of transport system facilities.

2. Problem statement

It is known that in practice, real signals are the sum of useful signals$X(t)$ and the noise$\mathrm{E}(t)$, i.e. $G(t) = X(t) + \mathrm{E}(t)$. In this case, the total noise $\mathrm{E}(t)$ is the sum of noise $\mathrm{E}_1(t)$ from the influence of external factors and the noise $\mathrm{E}_2(t)$ correlated with the useful signal, which occurs when the normal state of facilities of the transport system change, i.e. $\mathrm{E}(t)$=$\mathrm{E}_1(t)$+$\mathrm{E}_2(t)$. And traditionally it is assumed that the mathematical expectation of the noise $\mathrm{E}_1(t)$ is zero, $m_{\mathrm{E}_1} = 0$. And for the mathematical expectation of the correlated noise $\mathrm{E}_2(t)$ the approximate equality $m_{\mathrm{E}_2} \approx 0$ is valid. When there are malfunctions in facilities of the transport system, the value of the mathematical expectation of the total noise $\mathrm{E}(t)$ also begins to hover around zero, and the approximate equality$m_{\mathrm{E}} \approx 0$ is fulfilled. This means that the value of mathematical expectation of the total noise $\mathrm{E}(t)$ varies in some interval, depending on the degree of malfunctions and the rate of malfunction development, i.e.

$$m_{\mathrm{E}n} \leq m_{\mathrm{E}} \leq m_{\mathrm{E}v} \,, \tag{1}$$

where $m_{\mathrm{E}n}$, $m_{\mathrm{E}v}$ are the lower and upper limits of the variation.

At the same time, the appearance of the noise $\mathrm{E}_2(t)$ correlated with the useful signal $X(t)$ is reflected not only in the value of the variation interval of the mathematical expectation m_{E} , but also in the estimates of the correlation function $R_{GG}(\mu)$ of the noisy signal $G(t)$. It is known that the formula for calculating the estimate of the correlation function$R_{GG}(\mu)$ of the noisy signal $G(t)$ can be represented as [4-10]:

$$R_{GG}(\mu) = \sum_{i=1}^{N} G(i\Delta t)G\big((i+\mu)\Delta t\big) = R_{XX}(\mu) + \lambda_{GG}(\mu), \tag{2}$$

where $\lambda_{GG}(\mu)$ is the total error, which, given $R_{\mathrm{EE}}(\mu) = 0$ at $\mu \neq 0$, is

$$\lambda_{GG}(\mu) = \begin{cases} 2R_{X\mathrm{E}}(\mu) + R_{\mathrm{EE}}(0) & for\ \ \mu = 0 \\ R_{X\mathrm{E}}(\mu) + R_{\mathrm{E}X}(\mu) & for\ \ \mu \neq 0 \end{cases},$$

$R_{XX}(\mu) = \sum_{i=1}^{N} X(i\Delta t)X\big((i+\mu)\Delta t\big)$ is the autocorrelation function of the useful signal $X(t)$; $R_{X\mathrm{E}}(\mu) = \sum_{i=1}^{N} X(i\Delta t)\mathrm{E}\big((i+\mu)\Delta t\big)$, $R_{\mathrm{E}X}(\mu) = \sum_{i=1}^{N} \mathrm{E}(i\Delta t)X\big((i+\mu)\Delta t\big)$ are the cross-correlation functions between$X(t)$ and $\mathrm{E}(t)$; $R_{\mathrm{EE}}(\mu) = \sum_{i=1}^{N} \mathrm{E}(i\Delta t)\mathrm{E}\big((i+\mu)\Delta t\big)$ is the autocorrelation function of the noise $\mathrm{E}(t)$; $R_{\mathrm{EE}}(0) = D_{\mathrm{E}} = \sum_{i=1}^{N} \mathrm{E}(i\Delta t)\mathrm{E}(i\Delta t)$ are the variance of the noise $\mathrm{E}(t)$.

Thus, due to the contamination of the useful signals $X(t)$ with the noises $\mathrm{E}(t)$, estimates of correlation functions $R_{XX}(\mu)$ contain tangible errors $\lambda_{GG}(\mu)$. It follows that the inequality$R_{XX}(\mu) \neq R_{GG}(\mu)$, which arises due to the presence of the noise characteristics $R_{X\mathrm{E}}(\mu)$, $R_{\mathrm{E}X}(\mu)$, $R_{\mathrm{EE}}(0)$ in the estimates$R_{GG}(\mu)$, is true in the presence of malfunctions and damage. Therefore, it is not possible to draw adequate conclusions about the technical condition of the facilities of the transport system based on the estimates $R_{GG}(\mu)$. Since the noise $\mathrm{E}(t)$ cannot be isolated from the noisy signal (t) , the problem of extracting the information contained in the estimates of the noise characteristics $R_{X\mathrm{E}}(\mu)$, $R_{\mathrm{E}X}(\mu)$, $R_{\mathrm{EE}}(0)$ is obvious for the monitoring of the transport system.

Given the above, it is necessary to create algorithms and technologies to determine the estimates of the noise variance D_{E}, cross-correlation functions $R_{X\mathrm{E}}(\mu)$, $R_{\mathrm{E}X}(\mu)$ between the useful signal and the noise, as well as the variation interval of the mathematical expectation m_{E} of the total noise $\mathrm{E}(t)$.

3. Algorithms for analyzing the current condition of transport system facilities based on noise characteristics

As noted above, when there is a malfunction of facilities on the territory of the transport system, the noise $\mathrm{E}_2(t)$, correlated with the useful signal $G(t)$, appears. Therefore, starting from this point, the estimates of the cross-correlation functions $R_{X\mathrm{E}}(\mu)$, $R_{\mathrm{E}X}(\mu)$ and the correlation coefficient $r_{X\mathrm{E}}$ between the useful signal $X(t)$ and the total noise $\mathrm{E}(t) = \mathrm{E}_1(t) + \mathrm{E}_2(t)$ differ from zero [4-9]. This makes it possible to use the estimates $R_{X\mathrm{E}}(\mu)$, $R_{\mathrm{E}X}(\mu)$, $r_{X\mathrm{E}}$ as a carrier of diagnostic information of the early period of occurrence of malfunctions of facilities of the transport system. To calculate the above characteristics, we will use the formula for calculating the estimates of relay cross-correlation functions between the useful signal $X(t)$ and noise ` $\mathrm{E}(t)$ [4]:

$$R^{r}_{X\mathrm{E}}(\mu) = \sum_{i=1}^{N} sgnX(i\Delta t)\mathrm{E}\big((i+\mu)\Delta t\big), \tag{3}$$

where $gnX(i\Delta t) = \begin{cases} +1 & when \quad X(i\Delta t) > 0 \\ 0 & when \quad X(i\Delta t) = 0 \\ -1 & when \quad X(i\Delta t) < 0 \end{cases}$.

Obviously, to apply this formula, it is necessary to determine the samples of the noise $\mathrm{E}(i\Delta t)$ and the useful signal $X(i\Delta t)$, which cannot be measured directly or isolated from the noisy signal $G(t)$ [4]. Let us consider one of the possible variants of the approximate calculation of the estimates of the relay cross-correlation function $R^{r*}_{X\mathrm{E}}(\mu)$ between the useful signal $X(t)$ and the noise $\mathrm{E}(t)$ as a result of calculating the relay correlation function $R^{r}_{GG}(\mu)$ of the noisy signal $G(t)$ [4-9]:

$$R^{r}_{GG}(\mu) = \sum_{i=1}^{N} sgnG(i\Delta t)\mathrm{G}\big((i+\mu)\Delta t\big), \tag{4}$$

where $gnG(i\Delta t) = \begin{cases} +1 & when \quad G(i\Delta t) > 0 \\ 0 & when \quad G(i\Delta t) = 0 \\ -1 & when \quad G(i\Delta t) < 0 \end{cases}$.

It has been shown in [4-9] that the estimate of the relay cross-correlation function $R^{r*}_{X\mathrm{E}}(\mu\Delta t)$ at different time shifts between $X(i\Delta t)$ and $\mathrm{E}(i\Delta t)$ can be determined from the formula:

$$R^{r*}_{X\mathrm{E}}(\mu) = \frac{1}{N}\sum_{i=1}^{N} sgnG(i\Delta t)\left[\mathrm{G}\big((i+\mu)\Delta t\big) - 2\mathrm{G}\big((i+\mu+1)\Delta t\big) + \mathrm{G}\big((i+\mu+2)\Delta t\big)\right] \quad (5)$$

It is clear that in the normal state of facilities of the transport system due to the lack of correlation between $X(t)$ and $\mathrm{E}(t)$ the estimates of the relay cross-correlation function $R^{r}_{X\mathrm{E}}(\mu)$ between the useful signal and the noise will be close to zero. When various malfunctions and defects occur, the value of the estimate of the relay cross-correlation function $R^{r}_{X\mathrm{E}}(\mu)$ will vary depending on the degree of correlation between $X(t)$ and $\mathrm{E}(t)$. And the distinctive feature of this algorithm is that in the presence of correlation between$X(t)$ and$\mathrm{E}(t)$ the values of $R^{r*}_{X\mathrm{E}}(\mu)$ at different moments of time can be used as informative attributes of the early onset of malfunctions of transport system facilities, as well as the degree of their escalation into an emergency state.

In addition, to assess the early onset of malfunctions in facilities of the transport system, it is advisable to use estimates of the normalized cross-correlation function, as well as the correlation coefficient between the useful signal $X(t)$ and noise $\mathrm{E}(t)$.

In this case, based on formula (5), he normalized cross-correlation function can be calculated from the formula:

$$\rho^{*}_{X\mathrm{E}}(\mu) = \frac{R^{r*}_{X\mathrm{E}}(\mu)}{\sqrt{\frac{2}{\pi}}\cdot\sigma^{*}_{\mathrm{E}}}, \quad (6)$$

where the standard deviation σ^{*}_{E} of the noise is calculated from the expression [4-9]:

$$\sigma^{*}_{\mathrm{E}} = \begin{cases} \sqrt{R_{GG}(\mu=0) - 2R_{GG}(\mu=\Delta t) + R_{GG}(\mu=2\Delta t)} & \text{for the general case} \\ \sqrt{R_{GG}(\mu=0) - R_{GG}(\mu=\Delta t)} & \text{for the special case} \end{cases} \quad (7)$$

At the same time, it is known that the value of the normalized cross-correlation function at $\mu = 0$ is the correlation coefficient. Therefore, the value of the correlation coefficient between the useful signal $X(t)$ and the noise $\mathrm{E}(t)$ can be calculated from the expression:

$$r^{*}_{X\mathrm{E}} = \rho^{*}_{X\mathrm{E}}(0) = \frac{R^{r*}_{X\mathrm{E}}(0)}{\sqrt{\frac{2}{\pi}}\cdot\sigma^{*}_{\mathrm{E}}}. \quad (8)$$

Thus, algorithms have been developed for indicating the onset of malfunctions, damage and faults in facilities of the transport system, which are reduced to the calculation of the mean square deviation σ^{*}_{E} of the noise$\mathrm{E}(t)$, the relay cross-correlation function $R^{r*}_{X\mathrm{E}}(\mu)$ between $X(t)$ and $\mathrm{E}(t)$, the normalized cross-correlation function $\rho^{*}_{X\mathrm{E}}(\mu)$ and the correlation factor $r^{*}_{X\mathrm{E}}$ between the useful signal $X(t)$ and noise $\mathrm{E}(t)$.

4. Algorithms for analyzing the rate of change in the technical condition of transport system facilities

Below we propose algorithms for determining the rate of the development of malfunctions in transport system facilities using confidence interval (1) for the mathematical expectation of the noise $\mathrm{E}(t)$. It is known that the confidence interval for estimating the mathematical expectation of the noise with a known mean square deviation σ_{E} is calculated from the expression [8,9]:

$$\left(m_{\mathrm{E}} - z_p\cdot\frac{\sigma_{\mathrm{E}}}{\sqrt{N}};\ m_{\mathrm{E}} + z_p\cdot\frac{\sigma_{\mathrm{E}}}{\sqrt{N}}\right), \quad (9)$$

where N is the sample size; z_p is the critical value of the distribution, which can be found by setting a certain confidence probability $p = 1 - \alpha = \Phi(z)$; $\Phi(z)$ is the Laplace function. For example, to construct an interval with a 95% confidence level, we take $\alpha = 0.05$; then for probability p=0.95 we have $z_{0.95}$ =1.96.

From formula (9) it is obvious that determining the confidence interval for the mathematical expectation of the noise requires determining the mean square deviation $\sigma_{\mathrm{E}} = \sqrt{D_{\mathrm{E}}}$ of the noise $\mathrm{E}(t)$. Obviously, the mean square deviation σ_{E}^* of the noise $\mathrm{E}(t)$ can be calculated from expression (7). Then, taking into account expression (9) and the condition $m_{\mathrm{E}} \approx 0$, we can calculate the confidence interval for the mathematical expectation m_{E}^* of the noise:

$$\left(m_{\mathrm{E}} - z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}};\ m_{\mathrm{E}} + z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}\right) \tag{10}$$

or

$$m_{\mathrm{E}} - z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}} \le m_{\mathrm{E}}^* \le m_{\mathrm{E}} + z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}.$$

Since $m_{\mathrm{E}} \approx 0$, we get

$$-z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}} \le m_{\mathrm{E}}^* \le z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}. \tag{11}$$

Given that the expectation cannot be a negative number, the lower limit of the confidence interval is limited to zero and the upper bound to the number $z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}$. Hence,

$$0 \le m_{\mathrm{E}}^* \le z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}. \tag{12}$$

Thus, the lower bound of the confidence interval for the mathematical expectation of the noise $\mathrm{E}(t)$ is

$$m_{\mathrm{E}n}^* = 0, \tag{13}$$

and the upper bound:

$$m_{\mathrm{E}v}^* = z_p \cdot \frac{\sigma_{\mathrm{E}}^*}{\sqrt{N}}. \tag{14}$$

By constructing a confidence interval for the mathematical expectation of the noise $\mathrm{E}(t)$ at certain time instants, it is possible to determine the rate of the development of malfunctions of transport system facilities.

5. Technologies of analysis of serviceability of transport system facilities based on noise characteristics

To assess the technical condition of facilities of the transport system it is necessary to periodically calculate estimates of the relay and normalized cross-correlation functions, as well as correlation coefficients between the useful signal $\mathrm{X}(t)$ and the noise $\mathrm{E}(t)$ for each of the monitored parameters at different time instants $t_1, t_2, \cdots, t_k$. The obtained values of the estimates should be entered into the base of informative attributes:

$$TS1 = \begin{bmatrix} R_{X_1\mathrm{E}_1}^{r*}(\mu)_{t0} & R_{X_1\mathrm{E}_1}^{r*}(\mu)_{t1} & \cdots & R_{X_1\mathrm{E}_1}^{r*}(\mu)_{tk} \\ R_{X_2\mathrm{E}_2}^{r*}(\mu)_{t0} & R_{X_2\mathrm{E}_2}^{r*}(\mu)_{t1} & \cdots & R_{X_2\mathrm{E}_2}^{r*}(\mu)_{tk} \\ \cdots & \cdots & \cdots & \cdots \\ R_{X_n\mathrm{E}_n}^{r*}(\mu)_{t0} & R_{X_n\mathrm{E}_n}^{r*}(\mu)_{t1} & \cdots & R_{X_n\mathrm{E}_n}^{r*}(\mu)_{tk} \end{bmatrix} \tag{15}$$

$$TS2 = \begin{bmatrix} \rho_{X_1\mathrm{E}_1}^{*}(\mu)_{t0} & \rho_{X_1\mathrm{E}_1}^{*}(\mu)_{t1} & \cdots & \rho_{X_1\mathrm{E}_1}^{*}(\mu)_{tk} \\ \rho_{X_2\mathrm{E}_2}^{*}(\mu)_{t0} & \rho_{X_2\mathrm{E}_2}^{*}(\mu)_{t1} & \cdots & \rho_{X_2\mathrm{E}_2}^{*}(\mu)_{tk} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{X_n\mathrm{E}_n}^{*}(\mu)_{t0} & \rho_{X_n\mathrm{E}_n}^{*}(\mu)_{t1} & \cdots & \rho_{X_n\mathrm{E}_n}^{*}(\mu)_{tk} \end{bmatrix} \tag{16}$$

$$TS3 = \begin{bmatrix} r^*_{X_1\mathrm{E}_1-t0} & r^*_{X_1\mathrm{E}_1-t1} & \cdots & r^*_{X_1\mathrm{E}_1-tk} \\ r^*_{X_2\mathrm{E}_2-to} & r^*_{X_2\mathrm{E}_2-t1} & \cdots & r^*_{X_2\mathrm{E}_2-tk} \\ \cdots & \cdots & \cdots & \cdots \\ r^*_{X_n\mathrm{E}_n-t0} & r^*_{X_n\mathrm{E}_n-t1} & \cdots & r^*_{X_n\mathrm{E}_n-tk} \end{bmatrix}. \quad (17)$$

If all characteristics (15)-(17) of the relationship between the useful signal and the noise at the time instant t_0 are zero

$$R^{r*}_{X_i\mathrm{E}_i}(\mu)_{t0} = 0,$$

$$\rho^*_{X_i\mathrm{E}_i}(\mu)_{t0} = 0,$$

$$r^*_{X_i\mathrm{E}_i-t0} = 0 \, ,$$

then it indicates the absence of malfunctions, damage and faults of facilities of the transport system. Then for each parameter at the time instant t_0 using formulas (7), (12)-(14), the width of the confidence interval for the mathematical expectation of the noise $\mathrm{E}(t)$ at a constant level of significance and sample size is calculated:

$$0 \le m^*_{\mathrm{E-t0}} \le z_p \cdot \frac{\sigma^*_{\mathrm{E-to}}}{\sqrt{N}}. \quad (18)$$

Next, the set of possible values of the mathematical expectation of the noise at the time instant t_0, which fall within the constructed confidence interval, is compiled:

$$M^*_{\mathrm{E-t0}} = \{m^*_{\mathrm{E-t0}} | m^*_{\mathrm{En-t0}} \le m^*_{\mathrm{E-t0}} \le m^*_{\mathrm{Ev-t0}}\}$$

or

$$\overset{*}{M}_{\mathrm{E-t0}} = \{m^*_{\mathrm{E-t0}} | 0 \le m^*_{\mathrm{E-t0}} \le m^*_{\mathrm{Ev-t0}}\}. \quad (19)$$

In this case the difference of even one of the characteristics (15)-(17) of the relationship between the useful signal and the noise at the time instant t_1 , as well as subsequent time instants $t_2, \cdots, t_k$ from zero

$$R^{r*}_{X_i\mathrm{E}_i}(\mu)_{t1} \ne 0,$$

$$\rho^*_{X_i\mathrm{E}_i}(\mu)_{t1} \ne 0,$$

$$r^*_{X_i\mathrm{E}_i-t1} \ne 0$$

indicates the presence of correlation between the useful signal $X(t)$ and the noise $\mathrm{E}(t)$, resulting from the appearance of the noise $\mathrm{E}_2(t)$ correlated with the useful signal caused by changes in the normal state of facilities of the transport system.

Then to control the dynamics and rate of the development of malfunctions and failures at the instant t_1 the confidence interval

$$0 \le m^*_{\mathrm{E-t1}} \le z_p \cdot \frac{\sigma^*_{\mathrm{E-t1}}}{\sqrt{N}} \quad (20)$$

should be calculated, and a set of possible values of the mathematical expectation of the noise should be compiled:

$$M^*_{\mathrm{E-t1}} = \{m^*_{\mathrm{E-t1}} | m^*_{\mathrm{En-t1}} \le m^*_{\mathrm{E-t1}} \le m^*_{\mathrm{Ev-t1}}\}$$

or

$$\overset{*}{M}_{\mathrm{E-t1}} = \{m^*_{\mathrm{E-t1}} | 0 \le m^*_{\mathrm{E-t1}} \le m^*_{\mathrm{Ev-t1}}\}. \quad (21)$$

Then we find the difference between the sets of possible values of the mathematical expectation of the noise $M^*_{\mathrm{E-t1}}$ at the time instant t_1 and possible values of the mathematical expectation of the noise $M^*_{\mathrm{E-t0}}$ at the time instant t_0:

at $m^*_{\mathrm{Ev-t1}} > m^*_{\mathrm{Ev-t0}}$

$$M^*_{\mathrm{E-t0-t1}} = M^*_{\mathrm{E-t1}} \backslash M^*_{\mathrm{E-t0}} := M^*_{\mathrm{E-t1}} \cap \overline{M^*_{\mathrm{E-t0}}} = \{m^*_{\mathrm{E-t1}} | m^*_{\mathrm{E-t1}} \in M^*_{\mathrm{E-t1}} \ \textit{and} \ m^*_{\mathrm{E-t1}} \notin M^*_{\mathrm{E-t0}}\}, \quad (22)$$

that is, those values of the mathematical expectation, which are included in the set $M^*_{\mathrm{E-t1}}$, but not in the set $M^*_{\mathrm{E-t0}}$, and establish a correspondence between the value of the difference $M^*_{\mathrm{E-t0-t1}}$ and the degree of damage.

If after some time at the instant t_2 it is still not possible to eliminate the malfunction, and again the following inequalities are fulfilled:

$$R^{r*}_{X_i\mathrm{E}_i}(\mu)_{t2} \neq 0,$$

$$\rho^*_{X_i\mathrm{E}_i}(\mu)_{t2} \neq 0,$$

$$r^*_{X_i\mathrm{E}_i-t2} \neq 0,$$

then the confidence interval should be reconstructed:

$$0 \leq m^*_{\mathrm{E-t2}} \leq z_p \cdot \frac{\sigma^*_{\mathrm{E-t2}}}{\sqrt{N}}, \quad (23)$$

and a set of possible values of the mathematical expectation of the noise at the time instant t_2 should be compiled:

$$M^*_{\mathrm{E-t2}} = \{m^*_{\mathrm{E-t2}} | m^*_{\mathrm{En-t2}} \leq m^*_{\mathrm{E-t2}} \leq m^*_{\mathrm{Ev-t2}}\}$$

or

$$M^*_{\mathrm{E-t2}} = \{m^*_{\mathrm{E-t2}} | 0 \leq m^*_{\mathrm{E-t2}} \leq m^*_{\mathrm{Ev-t2}}\}. \quad (24)$$

After that we find the difference of the sets of possible values of the mathematical expectation of the noise $M^*_{\mathrm{E}-t2}$ at the time instant t_2 and possible values of the mathematical expectation of the noise $M^*_{\mathrm{E}-t1}$ at the time instant t_1:

at $m^*_{\mathrm{Ev-t2}} > m^*_{\mathrm{Ev-t1}}$

$$M^*_{\mathrm{E-t1-t2}} = M^*_{\mathrm{E-t2}} \backslash M^*_{\mathrm{E-t1}} := M^*_{\mathrm{E-t2}} \cap \overline{M^*_{\mathrm{E-t1}}} = \{m^*_{\mathrm{E-t2}} | m^*_{\mathrm{E-t2}} \in M^*_{\mathrm{E-t2}} \ \textit{and} \ m^*_{\mathrm{E-t2}} \notin M^*_{\mathrm{E-t1}}\}, \quad (25)$$

that is, those values of the mathematical expectation of the noise, which are included in the set $M^*_{\mathrm{E-t2}}$, but not included in the set $M^*_{\mathrm{E-t1}}$, and compare the differences $M^*_{\mathrm{E-t1-t2}}$ and $M^*_{\mathrm{E-t0-t1}}$. If the difference $M^*_{\mathrm{E-t1-t2}}$=$M^*_{\mathrm{E-t0-t1}}$, then the malfunction develops with uniform rate. If the difference $M^*_{\mathrm{E-t1-t2}}$>$M^*_{\mathrm{E-t0-t1}}$, the malfunction develops intensively, and if $M^*_{\mathrm{E-t1-t2}}$>>$M^*_{\mathrm{E-t0-t1}}$, the malfunction is transient. Then, depending on the degree of development of the malfunction, appropriate preventive or repair work should be carried out with or without shutting down the operation of the investigated facility of the transport system.

Thus, comparing the values of the differences of the sets of possible values of the mathematical expectation of the noise $M^*_{\mathrm{E-tm}}$ at the time instant t_m and possible values of the mathematical expectation of the noise $M^*_{\mathrm{E-t,m+1}}$ at the time instant t_{m+1}, provided that the characteristics of the relationship between the useful signal and the noise are different from zero

$$R^{r*}_{X_i\mathrm{E}_i}(\mu)_{tm} \neq 0,$$

$$\rho^*_{X_i\mathrm{E}_i}(\mu)_{tm} \neq 0,$$

$$r^*_{X_i\mathrm{E}_i-tm} \neq 0,$$

conclusions are drawn about the dynamics of the malfunction.

6. Results of the computational experiment on monitoring of malfunctions, damages and faults of transport system facilities and determining the rate of their development

To check the reliability of the algorithm for calculating the confidence interval for the mathematical expectation of the noise $\mathrm{E}(t)$ of the noisy signal $\mathrm{E}(t)$ we have conducted computational experiments using MATLABю Computational experiments are carried out as follows.

First, a useful signal $X(t)$ is formed. It is known that any stationary random process $X(t)$ over an infinite interval T can be approximated with any accuracy by a linear combination of harmonic oscillations with random amplitude and phase [6-11]. In the general form, the set of functions

$$X_k(t) = \sum_{\nu=1}^{n} \left(a_{\nu k} \cos\left(\frac{2\pi\nu}{T} t + \varphi_{1\nu k}\right) + b_{\nu k} \sin\left(\frac{2\pi\nu}{T} t + \varphi_{2\nu k}\right)\right)$$

describes a random process if the probability distribution functions of the coefficients $a_{\nu k}$, $b_{\nu k}$ and phases $\varphi_{1\nu k}$, $\varphi_{2\nu k}$ are known [10]. Therefore, when conducting computational experiments, a useful random signal

$$X(t) = 40 \cdot \sin\left(2\pi \frac{(k \cdot 0.2)^n}{T} + \varphi\right) + 100$$

is modeled as a perturbed harmonic discrete function with initial phase φ, which has a uniform probability distribution; where $k \in [0, K]$, $K = 300$; degree exponent $n = 1.5$; signal period $T = 100$; the initial phase φ is given as rand(size(k))*pi/3, where the function rand(size(k)) forms a vector commensurate with the vector k, whose elements are random variables distributed according to a uniform law in the interval (0, 1) [10].

It is assumed that the useful signal is a stationary ergodic process and $X(t)$ is one of its realizations.

The random number generator generates normally distributed noises $\mathrm{E}_i(t)$ with given values of mean square deviations $\sigma_{\mathrm{E}_i} = \sqrt{D_{\mathrm{E}_i}}$ and zero mathematical expectation $m_{\mathrm{E}_i} = 0$. It is assumed that these are the true noises at time instants t_i. For simplicity of the experiment for three time instants t_1, t_2 and t_3, three noises $\mathrm{E}_1(t)$, $\mathrm{E}_2(t)$, $\mathrm{E}_3(t)$ are generated with given values of mean square deviations σ_{E_1}=18.45, σ_{E_2}=23.06, σ_{E_3}=27.67 and zero mathematical expectation m_{E}. Then the noisy signals $\mathrm{G}_1(t)$=$X(t)$+$\mathrm{E}_1(t)$, $\mathrm{G}_2(t)$=$X(t)$+$\mathrm{E}_2(t)$, $\mathrm{G}_3(t)$=$X(t)$+$\mathrm{E}_3(t)$ are generated.

After that, the estimates of the mean square deviations of the noise are calculated using the proposed algorithm (7): $\sigma^*_{\mathrm{E}_1}$=19.193, $\sigma^*_{\mathrm{E}_2}$=23.267, $\sigma^*_{\mathrm{E}_3}$=27.441. From the above results, it is obvious that these values are almost the same as the given values σ_{E_1}=18.45, σ_{E_2}=23.06, σ_{E_3}=27.67. Therefore, the calculated estimates of $\sigma^*_{\mathrm{E}_1}$, $\sigma^*_{\mathrm{E}_2}$, $\sigma^*_{\mathrm{E}_3}$ can be used to determine the confidence interval for the mathematical expectation of the noise.

Then the upper bounds of the confidence interval for the mathematical expectation are calculated for all three noises $\mathrm{E}_1(t)$, $\mathrm{E}_2(t)$, $\mathrm{E}_3(t)$ from the expressions:

$$m^*_{\mathrm{E}v1} = z_p \cdot \frac{\sigma^*_{\mathrm{E}_1}}{\sqrt{N}},$$

$$m^*_{\mathrm{E}v2} = z_p \cdot \frac{\sigma^*_{\mathrm{E}_2}}{\sqrt{N}},$$

$$m^*_{\mathrm{E}v3} = z_p \cdot \frac{\sigma^*_{\mathrm{E}_3}}{\sqrt{N}},$$

and the confidence intervals at the time instants t_1, t_2 and t_3 are determined:

$$0 \le m^*_{E1} \le m^*_{E\nu 1},$$
$$0 \le m^*_{E2} \le m^*_{E\nu 2}$$
$$0 \le m^*_{E3} \le m^*_{E\nu 3}.$$

Thus, the following values of the upper bounds of the confidence interval have been obtained: $m^*_{E\nu 1}$=3.07, $m^*_{E\nu 2}$=3.6907, $m^*_{E\nu 3}$=4.4289, and the confidence intervals for the mathematical expectation of the noise have been constructed:

$$0 \le m^*_{E1} \le 3.07,$$
$$0 \le m^*_{E2} \le 3.69,$$
$$0 \le m^*_{E3} \le 4.42\ .$$

From the above results, it can be seen that the width of the confidence interval for the mathematical expectation of the noise increased with time $t_2 - t_1$ and $t_3 - t_2$. Based on the differences

$$M^*_{E2-E1} = M^*_{E2} \backslash M^*_{E1} := M^*_{E2} \cap \overline{M^*_{E1}} = \{m^*_{E2} | m^*_{E2} \in M^*_{E2}\ \ and\ \ m^*_{E2} \notin M^*_{E1}\},$$
$$M^*_{E3-E2} = M^*_{E3} \backslash M^*_{E2} := M^*_{E3} \cap \overline{M^*_{E2}} = \{m^*_{E3} | m^*_{E3} \in M^*_{E3}\ \ and\ \ m^*_{E3} \notin M^*_{E2}\},$$

that is, based on the differences 3.69-3.07 and 4.42-3.69 after appropriate training, the judgment is made about the dynamics of the development of malfunction, damage, faults in the facilities of the transport system. In this case, the mathematical expectation of the noise is zero and does not change over time, but the confidence interval changes. Therefore, the confidence interval is a more informative indicator of changes in the technical condition of transport system facilities.

Similar results have been obtained in other computational experiments. For example, we modeled a useful random signal

$$X(t) = 20 \cdot \cos\left(2\pi \frac{(k \cdot 0.5)^{n1}}{T} + \varphi_1\right) + 25 \cdot \sin\left(2\pi \frac{(k \cdot 1.5)^{n2}}{T} + \varphi_2\right) + 100$$

as a perturbed harmonic discrete function with amplitudes and initial phases φ_1, φ_2, which have uniform probability distribution (or with uniform probability density function); where $k \in [0, K]$, K =200, degree indices $n1 = 1.5$, $n2 = 0.5$; signal period $T = 100$; amplitudes are given as rand(size(k)), rand(size(k)); initial phases φ_1, φ_2 are given as rand(size(k))*pi/3, rand(size(k))*pi/3 [10]. The noises are generated similarly to those of the first experiment with mean square deviations σ_{E_1}=22.677, σ_{E_2}=27.21, σ_{E_3}=31.75. The results of computational experiments are summarized in Table 1.

Table 1

Results of the experiments to determine the confidence interval for the mathematical expectation of the noise at different instants of time

	σ_{E_1}	$\sigma^*_{E_1}$	$\Delta\sigma_{E_1}$,%	σ_{E_2}	$\sigma^*_{E_2}$	$\Delta\sigma_{E_2}$,%	σ_{E_3}	$\sigma^*_{E_3}$	$\Delta\sigma_{E_3}$,%
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	22.677	23.215	2.32	27.21	27.46	0.89	31.75	31.75	0.01
	$m_{E\nu 1}$	$m^*_{E\nu 1}$	$\Delta m_{E\nu 1}$,%	$m_{E\nu 2}$	$m^*_{E\nu 2}$	$\Delta m_{E\nu 2}$,%	$m_{E\nu 3}$	$m^*_{E\nu 3}$	$\Delta m_{E\nu 3}$,%
2	4.44	4.55	2.37	5.33	5.38	0.8967	6.22	6.22	0.01

Thus, from the results of multiple computational experiments it is obvious that the increase in the width of the confidence interval for the mathematical expectation of the noise is a rather effective informative attribute for controlling the occurrence of malfunctions, damage and faults of transport system facilities and for determining the rate of their development at an early stage.

7. Conclusion

Analysis shows that using the algorithms for calculating the characteristics of the relationship between the useful signal and the noise can identify the early stage of the occurrence of malfunctions, damage and faults in facilities of the transport system. At the same time, the developed algorithms for constructing the confidence interval for the mathematical expectation of the noise allow us to determine the dynamics of malfunction development. In this case, the width of the confidence interval for the mathematical expectation of noise, which changes over time in accordance with the development of the defect, is taken as an informative feature.

Using the developed algorithms and technologies in the systems of monitoring and warning of malfunctions, damage and faults in facilities of the transport system allows ensuring safe conditions of their operation, reliability, safety and comfort of passenger service, reliability of cargo and passenger transportation, significantly reduce the risk of transport accidents with severe consequences, entailing damage to property, the environment, life and health of citizens.

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