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Optimal selection of variants for construction projects according to allocated capital investment

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A B S T R A C T

The article considers the problem of putting a certain number of construction projects into operation. It is assumed that a corresponding number of variants (crews) can be used to put each project into operation. It is required that for each construction project such variants were selected and used that the total cost required for them not exceed the allocated funds, that only one variant be used for each project, and the income received be maximum. A mathematical model of this problem is built and an approximate solution method of is developed.

1. Introduction

Suppose that for a certain planning period there are to be built and completed n construction projects, and there are possible m_i variants (crews) carrying out construction and completion of the i-th construction project$(i = \overline{1,n})$. Let us denote the income (effect) from the application of j-th variant $j = \overline{1,m_i}$ for i-th construction project $(i = \overline{1,n})$ by c_{ij} $(i = \overline{1,n}, j = \overline{1,m_i})$. Let the sum of required funds from the total allocated capital investments K when applying j-th variant $(j = \overline{1,m_i})$ for i-th construction project $(i = \overline{1,n})$ will be k_{ij}. By virtue of the nature of the problem, the natural conditions $c_{ij} \geq 0,\ k_{ij} \geq 0,\ \ (i = \overline{1,n};\ j = \overline{1,m_i})$ should be satisfied. It is clear that only one variant should be chosen for all construction projects being put into operation. Otherwise, the construction project cannot be put into operation. At the same time, it is necessary to select and apply variants for the construction of all construction projects in the plan such that the amount of funds expended on these variants not exceed the total allocated capital investments K and, the total income (effect) received from the projects built according to the selected variants within the given capital investments be maximum.

2. Model description and problem statement

To build a mathematical model of this problem, we assume the following unknowns x_{ij}.

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$$x_{ij} = \begin{cases} 1, if\ for\ i-th\ object\ (i=\overline{1,n})\ \text{variant}\ j-\text{ci}\ (j=\overline{1,m_i}) \\ \qquad is\ selected, \\ \quad 0, \qquad otherwise\ . \end{cases} \tag{1}$$

Income from projects built using the variants selected under these designations may amount to up to

$$\sum_{i=1}^{n}\sum_{j=1}^{\mathrm{m}_i} c_{ij}x_{ij}.$$

The amount of funds required for the variants must not exceed the total allocated capital investment K. Then the following inequality must be satisfied:

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} k_{ij}x_{ij} \le K\ . \tag{2}$$

The problem requires that only one option be selected for each construction project. The following system of equations ensures that this condition is satisfied.

$$\sum_{j=1}^{m_i} x_{ij} = 1\ (i=\overline{1,n}). \tag{3}$$

Thus, the mathematical model of the problem will look as follows.

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} c_{ij}x_{ij} \to max, \tag{4}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i} k_{ij}x_{ij} \le K\ , \tag{5}$$

$$\sum_{j=1}^{m_i} x_{ij} = 1\ (i=\overline{1,n}), \tag{6}$$

$$x_{ij} = 1 \vee 0\ \ (i=\overline{1,n}, j=\overline{1,m_\iota}). \tag{7}$$

Here $c_{ij} \ge 0,\ k_{ij} \ge 0,\ \ (i=\overline{1,n};\ j=\overline{1,m_i})\ k>0$ are prescribed constant numbers.

As can be seen, model (4)-(7) is a Boolean programming problem with special constraint conditions [1, 2]. Since this is an NP-complete problem [3], the known methods for finding its optimal solution [1, 4] cannot work in real time. Therefore, in this study, we developed a special approximate method for solving problem (4)-(7).

Note that the existence of a solution to the problem under consideration is unknown. The following theorem serves as a criterion for the existence of a solution to the problem under consideration.

Theorem: A necessary and sufficient condition for the solution of problem (4)-(7) must satisfy the inequality

$$\sum_{i=1}^{n}\{\min_{1\le j\le m_i} k_{ij}\} \le K. \tag{8}$$

Proof: Necessity.

Let us assume that the problem has a solution. That is, from the number of possible variants for each construction project it is possible to choose a variant such that the sum of funds k_{ij} required by these variants selected for all construction projects does not exceed the total amount of allocated capital investments K. That is, for these k_{ij} the following condition must be fulfilled

$$\sum_{i=1}^{n} k_{ij} \leq K \quad \left(j = \overline{1, m_i}\right). \tag{9}$$

That means that the minimum of k_{ij} for j will also be less than K. In other words, the amount of funds corresponding to the variants will also not exceed the total allocated capital investment K (the amount of funds corresponding to the minimum-cost variants will also be less than K). Therefore, condition (8) will be fulfilled.

This proves the necessity condition.

Sufficiency.

Suppose that condition (8) is satisfied. Let us prove that the problem has a solution.

The fulfillment of condition (8) means that there exist such variants corresponding to certain k_{ij} that make it possible to build all construction projects in the plan. This means that there is a solution to the problem. That is, the sum of funds corresponding to the variants requires minimum cost of construction and completion of all construction projects in the plan does not exceed the allocated total capital investment K.

Thus, the sufficiency condition and the theorem are proved.

3. Solution method

First of all, note that the optimal solution of problem (4)-(7) is a binary vector X such that satisfies constraints (5)-(7) and gives the maximum value of projective function (4).

This problem can be solved using the well-known branch and bound method. However, solving the problem under consideration with this method is disadvantageous in terms of time spent. The time it takes to solve a problem by this method increases exponentially depending on the number of unknowns, and as a result, the search for a solution requires unrealistic time.

Therefore, an efficient approximate method for solving problem (4)-(7) is proposed in this study. First, to find out whether the problem has a solution, the condition

$$\sum_{i=1}^{n} \{\min_{1 \leq j \leq m_i} k_{ij}\} \leq K$$

If this condition is satisfied, i.e. if there is a solution to the problem, the ratios $\frac{c_{ij}}{k_{ij}}$ $(i = \overline{1, n}, j = \overline{1, m_i})$ are calculated to find the profit per unit of cost required for each variant. First, let us assume the sets $I \coloneqq \{1,2, \dots, n\}$, $J \coloneqq \{1,2, \dots, m_i\}$. The found ratios $\frac{c_{ij}}{k_{ij}}$ are arranged in descending order by groups consisting of variants corresponding to each construction project. That is, for $\forall i$ $(i = \overline{1, n})$

$$\frac{c_{ij}}{k_{ij}} \geq \frac{c_{ij+1}}{k_{ij+1}}, j = \overline{1, m_\iota - 1}\,. \tag{10}$$

In other words,

$$\frac{C_{11}}{k_{11}} \geq \frac{C_{12}}{k_{12}} \geq \cdots \frac{C_{1m_1}}{k_{1m_1}}$$
$$\frac{C_{21}}{k_{21}} \geq \frac{C_{22}}{k_{22}} \geq \cdots \frac{C_{2m_2}}{k_{2m_2}},$$
....
....
....
$$\frac{C_{n1}}{k_{n1}} \geq \frac{C_{n2}}{k_{n2}} \geq \cdots \frac{C_{nm}}{k_{nm}}.$$

In this order, the ratios $\frac{c_{ij}}{k_{ij}}$, ranked first in each group, show the best variant available for each construction project. Therefore, for each $j \in J$ we choose the project No i_* from the ratio

$$\max_{i \in I} \left\{\frac{c_{ij}}{k_{ij}}\right\} = \frac{c_{i_* j_*}}{k_{i_* j_*}} \tag{11}$$

In the following steps, we no longer look at the project No i_*, because it is necessary to take into account condition (6). If in the groups consisting of variants there are several ratios whose denominators are equal to the minimal number of k_{ij} then the one that has the largest copy among these ratios is kept, the other ratios are discarded and the number of variants is reduced.

If $k_{i_* j_*} \leq K$e, then we must take $K \coloneqq K - k_{i_* j_*}$: $I \coloneqq I/\{i_*\}$ $J \coloneqq J/\{j_*\}$. $x_{i_* j_*} \coloneqq 1$, otherwise we write $x_{i_* j_*} \coloneqq 0$ $J \coloneqq J/\{j_*\}$ and continue the process of constructing the solution. That is, from relation (11) we find again the current numbers j_*, i_*. If $k_{i_* j_*} \leq K$ for these numbers i_*, j_*, we again take $x_{i_* j_*} \coloneqq 1$, $K \coloneqq K - k_{i_* j_*}$, $I \coloneqq I/\{i_*\}$ and $J \coloneqq J/\{j_*\}$ and continue the solution process in the above manner. Otherwise, i.e., if $k_{i_* j_*} > K$, we write $J \coloneqq J/\{j_*\}$, the solution process continues according to criterion (11).

The solution process continues until $I = \emptyset$, when all construction projects have been considered.

4. Conclusion

In this work, a mathematical model of putting a certain number of construction projects into operation during a given planning period is built. It is taken into account that different variants can be used for putting each construction project into operation. The model takes into account that for each project such variants should be selected and applied that their costs not exceed the allocated funds, only one option be used for each project, and the income from the use of these projects be maximum.

The mathematical model of these problems is obtained as a two-index Boolean programming problem with special constraint conditions. An approximate method for solving this problem is developed.

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