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Innovative approximate solution methods and error estimation for the knapsack problem

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ABSTRACT

A new innovative approximate solution method of the knapsack problem is developed. First, any approximate solution to the problem is found in the known manner. After that, the process of improving that solution is built successively. Here, a neighborhood of the fractional-valued coordinate in the optimal solution of the continuous problem is found by a certain rule, and a better solution is selected by replacing the units located in this neighborhood with zeros and then replacing the zeros with units.Using the proposed method, a numerical example is solved and extensive computational experiments are conducted on random problems of different dimensions.

1. Introduction

Consider the following well-known knapsack problem:

$$\sum_{j=1}^{n} c_j x_j \to \max, \tag{1}$$

$$\sum_{j=1}^{n} a_j x_j \le b, \tag{2}$$

$$x_j = 0 \vee 1,\ (j = \overline{1,n}). \tag{3}$$

Here c_j, a_j, $(j = \overline{1,n})$ and b are specified integers. Without violating generality, we can assume that these numbers are positive integers and the following relations hold.

$$\frac{c_1}{a_1} \ge \frac{c_2}{a_2} \ge \cdots \ge \frac{c_k}{a_k} \ge \cdots \ge \frac{c_n}{a_n}\ . \tag{4}$$

Note that relation (4) is sometimes written as follows:

$$\frac{c_j}{a_j} \ge \frac{c_{j+1}}{a_{j+1}}\ ,\quad (j = \overline{1, n-1}).$$

As is known, when replacing condition (3) in problem (1)-(3) $0 \le x_j \le 1$, with the condition $(j = \overline{1,n})$, its optimal solution $\bar{X} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$ is found in the following way [1].

If for every number $j = 1,2, \dots . n$

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$$a_j \leq b - \sum_{i=1}^{j-1} a_i \bar{x}_i,$$

then $\bar{x}_j := 1$, otherwise, i.e., if

$$a_j > b - \sum_{i=1}^{j-1} a_i \bar{x}_i,$$

then we take

$$\bar{x}_j := \left(b - \sum_{i=1}^{j-1} a_i \bar{x}_i \right) \Big/ a_j$$

and keep the last number j in mind. Suppose this is the k-th number, that is, $k := j$. For subsequent numbers $j = k + 1, k + 2, \dots, n$, we take that $\bar{x}_j := 0$.

Note that we can write this rule briefly in the form of the following formula: for every number j, $(j = \overline{1,n})$

$$\bar{x}_j := \begin{cases} 1, & if \quad a_j \leq b - \sum_{i=1}^{j-1} a_i \bar{x}_i \ , \\ \left(b - \sum_{i=1}^{j-1} a_i \bar{x}_i \right) \Big/ a_j, & if \quad a_j > b - \sum_{i=1}^{j-1} a_i \bar{x}_i \ , \ (k := j), \\ 0, & when \quad j = k + 1, k + 2, \dots, n. \end{cases}$$

As a result, we get the solution

$$\bar{X} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k, \dots, \bar{x}_n) = \left(\underbrace{1,1, \dots ,1,}_{k-1} \frac{\alpha}{\beta}, 0, \dots ,0 \right). \qquad (5)$$

Then the value $\bar{f}$ of function (1) corresponding to solution (5) in problem (1)-(3) is

$$\bar{f} = \sum_{j=1}^{n} c_j \bar{x}_j$$

If $\bar{x}_k = 1$ or $\bar{x}_k = 0$, then solution (5) is also the optimal solution of problem (1)-(3), and in this case the solution process ends.

Note that any $X^t = (x_1^t, x_2^t, \dots, x_n^t)$ approximate solution to problem (1)-(3) is found as follows according to the well-known rule [2].

For each number $j = 1,2, \dots, n$, $x_j^t := 1$, we assume that the condition

$$a_j \leq b - \sum_{i=1}^{j-1} a_i x_i^t$$

is satisfied. Otherwise, i.e., if the condition

$$a_j > b - \sum_{i=1}^{j-1} a_i x_i^t$$

is satisfied, we take that $x_j^t := 0$. As a result, we get the approximate solution the problem (1)-(3):

$$X^t = (x_1^t, x_2^t, \dots, x_n^t). \qquad (6)$$

Then the value of function (1) corresponding to this solution is

$$f^t = \sum_{j=1}^{n} c_j x_j^t.$$

It is clear that $f^t \leq \overline{f}$. Or rather, it should be $f^t \leq f^* \leq \overline{f}$. Here, the number f^* is the maximum value of function (1) in problem (1)-(3).

Note that in rare cases, the solution $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$ of (6) coincides with or is close to the optimal solution of problems (1)-(3). Numerous experiments show that the coordinates x_j^* of the optimal solution and the coordinates x_j^t of the approximate solution differ within a certain neighborhood from the coordinate found in formula (5). Because the ratios c_j/a_j for the numbers $\bar{x}_k = \frac{\alpha}{\beta}j$ in that neighborhood are close to each other. If we consider that there are only units on the left-hand side of that k-th coordinate, and only zeros on the right-hand side, and the coordinates of the optimal solution differ here, then a better solution can be obtained by replacing certain units on the left side of the k-th coordinate in formula (5) with zero, and a certain number of zeros on the right side of that coordinate alternately with units. But in this case, replacing all the units in formula (5) with zero, or all the zeros with unit, would require a lot of extra operations.

On the other hand, if we know which coordinates are "1" or "0" in the optimal solution, it is clear that changing those coordinates will not give a good solution. At the same time, numerous experiments show that certain successive coordinates at the beginning and end of the solution (5) found under condition (4) coincide with the optimal solution. Therefore, we need to find the minimum number of units and zeros in the optimal solution of problem (1)-(3). Suppose that the minimum number of units in the optimal solution is $n(1)$, and the minimum number of zeros is $n(0)$. Then it is clear that it is not necessary to change the coordinates of the first $n(1)$ and the last $n(0)$ in formula (5). The order of finding $n(1)$ and $n(0)$ will be shown in the following paragraphs.

It should be noted that the problem of finding the nearest neighborhood of the k-th coordinate found by formula (5) and changing the coordinates there was considered in works [3-5].

For example, in [3], the neighborhood $[k-p,\ k+p]$ of the k-th coordinate is selected. Here p is found from the condition

$$p = arg\left\{\max_i\left(\frac{c_{k-i}}{a_{k-i}} - \frac{c_{k+i}}{a_{k+i}}\right) \leq \delta\right\}.$$

(δ is a specified integer). It is clear that if the found number k is close to the beginning or the end of solution (5), then the interval $[k-p,\ k+p]$ cannot be used. In [4, 5], the interval $[\delta_k^1, \delta_k^0]$ of the k-th coordinate was chosen as the nearest neighborhood.

Here $\delta_k^1 = \left[k \cdot \frac{q}{100}\right]$, $\delta_k^0 = \left[(n-k) \cdot \frac{q}{100}\right]$, and q shows the number of units and zeros in solution (5) as a percentage. Of course, when applying such a rule, choosing q can create uncertainty. Indeed, if q is too large or too small, the applied method may lose adequacy. This property was revealed during the experiments conducted in [5].

Therefore, in this study, we give the rule for finding the minimum number of units $n(1)$ and the minimum number of zeros $n(0)$ in the optimal solution of problem (1)-(3). Here, there is no need to change the unit of the first $n(1)$ and the zero of the last $n(0)$ in the solution process.

Obviously, such an algorithm is general, requires fewer operations, can provide a better solution, and has important applications. Therefore, we will call the solution found by this rule an innovative approximate solution.

2. Problem statement

First, we find the minimum numbers of units $n(1)$ and zeros $n(0)$, respectively, in the optimal solution to problem (1)-(3). Because replacing the first $n(1)$ number of units and the last $n(0)$ number of zeros in the solution (5) with zeros and ones, respectively, cannot give a better solution. This is because those units and zeros are generally the coordinates of the optimal solution. Of course, changing those coordinates means changing the optimal solution. Therefore, no better solution can be found.

For this purpose, consider the following problems.

$$\sum_{j=1}^{n} x_j \rightarrow min \,, \tag{7}$$

$$\sum_{j=1}^{n} a_j x_j \leq b, \tag{8}$$

$$\sum_{j=1}^{n} c_j x_j \geq f^t, \tag{9}$$

$$x_j = 1 \vee 0, \quad j = (\overline{1,n}) \;. \tag{10}$$

and

$$\sum_{j=1}^{n} x_j \rightarrow max \;\;, \tag{11}$$

$$\sum_{j=1}^{n} a_j x_j \leq b, \tag{12}$$

$$\sum_{j=1}^{n} c_j x_j \geq f^t, \tag{13}$$

$$x_j = 1 \vee 0, \quad j = (\overline{1,n})\,. \tag{14}$$

It is important to note that in problem (7)-(10) the value of function (7) gives the number $n(1)$.

Assume that we have found the value $\bar{n}$ of function (11) by solving problem (11)-(14). It is clear that $\bar{n}$ is the maximum number of units in the optimal solution of problem (1)-(3). Then it is clear that the minimal number of zeros in the optimal solution is equal to $n - \bar{n}$. So, we can take that

$$n(0) = n - \bar{n}\,. \tag{15}$$

Thus, by solving problems (7)-(10) and (11)-(14), we find the corresponding numbers that indicate the minimum number of units $n(1)$ and zeros $n(0)$ in the optimal solution.

As can be seen, (7)-(10) and (11)-(14) are Boolean programming problems with two constraints and mixed inequalities. Finding the optimal solution to this problem is one of "difficult to solve" problems. Therefore, if we replace condition (10) in problem (7)-(10) and condition (14) in problem (11)-(14) with the condition $0 \leq x_j \leq 1$, $(j = \overline{1,n})$ to estimate $n(1)$ and $n(0)$, we will get a well-known linear programming problem. By solving these problems by known methods (for example, by the simplex method), we will find the corresponding $\underline{n(1)}$ and $\underline{n(0)}$.

We prove the following theorem.

Theorem: The inequalities $\underline{n(1)} \leq n(1)$ and $\underline{n(0)} \leq n(0)$ are true for the minimal number of units $n(1)$ and the minimal number of zeros $n(0)$ in the optimal solution of problem (1)-(3).

Proof. Denote the set of possible solutions of problem (7)-(10) by At and, replacing condition (10) in that problem with $0 \leq x_j \leq 1$, denote the set of possible solutions of the obtained problem by B. It is clear that $A \subset B$. Because the set $0 \leq x_j \leq 1$, $j = (\overline{1,n})$ is wider than the set $x_j = 1 \vee 0$, $j = (\overline{1,n})$. Then it is clear that for a certain function $f(x)$, we should have

$$\min_{x \in B} f(x) \leq \min_{x \in A} f(x).$$

It follows directly from this relation that $\underline{n(1)} \leq n(1)$.

On the other hand, if we denote the set of possible solutions of problem (11)-(14) by A, and, replacing the condition (14) in that problem with $0 \leq x_j \leq 1$, $j = (\overline{1,n})$, we denote the set of possible solutions by B, it is clear that $A \subset B$. Then for a certain function $f(x)$, the relation

$$\max_{x \in A} f(x) \leq \max_{x \in B} f(x)$$

is true. If we apply this principle to problem (11)-(14) and replace condition (14) with $0 \leq x_j \leq 1$, $j = (\overline{1, n})$, applying it to the obtained problem, we get the maximum number of units in the optimal solution of problem (1)-(3) as $\overline{n(1)}$ and $\overline{\overline{n(1)}}$, respectively. Here, $\overline{n(1)} \leq \overline{\overline{n(1)}}$. The minimum number of zeros in the optimal solution of problem (1)-(3) is $n - \overline{n(1)}$ or $n - \overline{\overline{n(1)}}$, respectively. Obviously, $n(0) = n - \overline{n(1)} \geq n - \overline{\overline{n(1)}}$ has to be satisfied. Since finding the number $\overline{n(1)}$ is related to problem (11)-(14), which is difficult to solve, $n(0) \geq n - \overline{\overline{n(1)}}$ can be accepted as the minimal number of zeros. If we take the notation $n - \overline{\overline{n(1)}} = \underline{n(0)}$, the theorem is completely proved.

Note that using this theorem, there is no need to change the first number $n(1)$ of units and the last number $n(0)$ of zeros of solution (5).

3. Solution method

Note that finding $\underline{n(1)}$ and $\underline{n(0)}$ can also cause certain difficulties, because a linear programming problem with the condition of inequalities of two different signs has to be solved.

On the other hand $n(0)$, since we want to evaluate the numbers of and , it can be done in a simpler way as follows.$n(1)$

On the other hand, since we want to estimate the numbers $n(0)$ and $n(1)$, it can be done in a simpler way as follows.

It is clear that the value given to the function (1) of the optimal solution of the problem (1)-(3) should not be smaller than the number we found above. I mean

It is clear that the value of the optimal solution of problem (1)-(3) that affords to function (1) should not be smaller than the number f^t we found above. I.e., it should satisfy the relation

$$\sum_{j=1}^{n} c_j x_j \geq f^t.$$

Therefore, the minimal number of units and zeros should be found for x_j-s $j = \overline{1, n})$ satisfying this condition. For this purpose, without violating the generality, assume that the coefficients c_j, ($j = \overline{1, n}$) are arranged as follows.

$$c_1 \geq c_2 \geq \cdots \geq c_{n-1} \geq c_n$$

Then we find $n_1$from the following relation.

$$\textstyle\sum_{j=1}^{n_1} c_j \leq f^t \leq \sum_{j=1}^{n_1+1} c_j \tag{16}$$

It is clear that the following relation is true for the found n_1.

$$n(1) \geq \underline{n(1)} \geq n_1$$

In our experiments, for simplicity, we will take the minimum number of units in the optimal solution as n_1.

n_0, which is close to the minimum number of zeros in the optimal solution $n(0)$, can be found in a simpler way as follows. For this, without violating generality, assume that the coefficients a_j of inequality (2) are arranged in the increasing order. That is, the relation $a_1 \leq a_2 \leq \cdots \leq a_n$ is satisfied. Then, in order to find the maximum number of units in the optimal solution, we need to solve problem (11)-(13) without considering inequality (13). In this case, the maximum number of units $\bar{n}_1$ in the optimal solution can be found from the following relation.

$$\sum_{j=1}^{\bar{n}_1} a_j \le b < \sum_{j=1}^{\bar{n}_1+1} a_j$$

Note that $\bar{n}_1$ satisfying this relation is the maximum number of units in the optimal solution. It is clear that $\bar{n}_1 > \overline{n}$. Then according to formula (15), for the minimum number of zeros $n(0)$ in the optimal solution, we take the relation

$$n(0) = n - \overline{n} \ge n - \bar{n}_1$$

If we take the notation $n_0 = n - \bar{n}_1$, the found n_0 indicates the minimum number of zeros in the optimal solution of problem (1)-(3). Rather, we get that $n(0) \ge n_0$.

Thus, in the manner shown above, we can easily find that the optimal solution of problem (1)-(3) must have at least n_1 units and at least n_0 zeros.

Therefore, we should keep the first $n(1)$ number of units and the last $n(0)$ number of zeros in solution (5) as they are. We keep in mind the biggest of the solutions that give to function (1) value greater than the number f^t from among the possible solutions obtained by replacing the units with zeros in the remaining coordinates, and the zeros with units alternately. We will call the solution last saved in this manner the innovative approximate solution of problem (1)-(3). Because the last solution of problem (1)-(3) in the NP-complete class is no worse than the solution found by the classical method, it is found by a simple rule, and it has important practical significance in terms of application.

We will now write the algorithm of the solution of the problem.

ALGORITHM

Step 1. Enter quantities n, b, c_j, a_j $(j = \overline{1, n})$ and take $r \coloneqq 0$ and $bb \coloneqq b$.

Step 2. For each $j, (j = 1,2, \dots, n)$, find the solution $\bar{X} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n, \dots, \bar{x}_k)$ from the formula

$$\bar{x}_j := \begin{cases} 1, \qquad if \quad a_j \le b - \sum_{i=1}^{j-1} a_i \bar{x}_i \ , \\ \left(b - \sum_{i=1}^{j-1} a_i \, \bar{x}_i \right) \Big/ a_j \, , if \quad a_j > b - \sum_{i=1}^{j-1} a_i \, \bar{x}_i \ , \quad (k \coloneqq j), \\ 0, \qquad when \, j = k+1, k+2, \dots, n. \end{cases}$$

note the number k.

Step 3. If $\bar{x}_k = 1$ or $\bar{x}_k = 0$, the solution $\bar{X} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$ is the optimal solution to (1)-(3). Calculate

$$f^* \coloneqq \sum_{j=1}^{n} c_j \, \bar{x}_j$$

print $X^* = (x_1^*, \ x_2^*, \dots, x_n^*)$, f^*, go to step 17.

Step 4. Find the approximate solution $X^t = (x_1^t, \ x_2^t, \dots, x_n^t)$as follows. For each $(j = 1,2, \dots, n)$

$$x_j^t = \begin{cases} 1, & if\ a_j \leq b - \sum_{i=1}^{j-1} a_i x_i^t, \\ 0, & otherwise. \end{cases}$$

Step 5. Find $\bar{f} = \sum_{j=1}^{n} c_j \bar{x}_j$ and $f^t = \sum_{j=1}^{n} c_j\, x_j^t$ and take that $f^{it} \coloneqq f^t$.

Step 6 . The coefficients $c_j, (j = \overline{1,n})$ should be arranged as $c_1 \geq c_2 \geq \cdots \geq c_n$. From the relation

$$\sum_{j=1}^{n_1} c_j \leq f^t \leq \sum_{j=1}^{n_1+1} c_j$$

find n_1.

Step 7 . The coefficients $a_j, (j = \overline{1,n})$ should be arranged as $a_1 \leq a_2 \leq \cdots \leq a_n$. From the relation

$$\sum_{j=1}^{\bar{n}_1} a_j \leq b < \sum_{j=1}^{\bar{n}_1+1} a_j$$

find $\bar{n}_1$, and then $n_0 = n - \bar{n}_1$.

Step 8. Take $x_k^t \coloneqq 1;\ \ b \coloneqq bb - a_k$.

Step 9 . For $j = 1,2, \dots, n$ and $j \neq k$

$$x_j^t = \begin{cases} 1, & if\ a_j \leq b - \sum_{i=1}^{j-1} a_i x_i^t \\ 0, & otherwise\ . \end{cases} \tag{18}$$

Step 10 . Calculate $f_1^t = \sum_{j=1}^{n} c_j x_j^t$.

Step 11 . If $f_1^t > f^{it}$, then $f^{it} := f_1^t, x_j^{it} \coloneqq x_j^t,\ \ (j = \overline{1,n})$

Step 12 . If r = 1, skip to step 15.

Step 13 . $k := k + 1$; if $k > n - n_0$, go to step 14, otherwise go to step 8.

Step 14 .$k := kk;\ \ \ r \coloneqq 1;$

Step 15 . $k := k - 1$;if $k < n_1$yes go to step 16 otherwise $x_k^t \coloneqq 0;\ \ b \coloneqq bb$accept and go to step 9

Step 16 . Print f^{it} ; $\ X^{it} = (x_1^{it}, x_2^{it}, \dots, x_n^{it})$

Step 17 . Stop.

Let us solve the following example using this algorithm.

$$22x_1 + 13x_2 + 21x_3 + 11x_4 + 16x_5 + 15x_6 + 12x_7 + 7x_8 + x_9 + x_{10} \to max$$

$$3x_1 + 2x_2 + 4x_3 + 3x_4 + 5x_5 + 6x_6 + 5x_7 + 4x_8 + 2x_9 + 5x_{10} \leq 15$$

$$x_j = 1 \vee 0\ ,\ \ (j = \overline{1,10})$$

Here, relation (4) has been satisfied. In this case, solution (5) is as follows.

$$\bar{X} = \left(1,1,1,1, \tfrac{3}{5}, 0,0,0,0,0\right) \tag{19}$$

Obviously, $k = 5$. Then $\bar{f} = 76{,}6$. On the other hand, according to formula (6), we get

$$X^t = (1,1,1,0,0,0,0,1,0)$$

and $f^t = 68$.

It is clear that in the problem under consideration, for the optimal value f^* of the objective function $68 \leq f^* \leq 76$.

We can find the minimum number of units n_1 and the minimum number of zeros n_0 in the optimal solution of the considered problem according to formulas (16) and (17). Then we get $n_1 = 3$ $n_0 = 4$.

Based on the above algorithm, we first construct new solutions by replacing zeros with "1". Then assuming $x_6^t = 1$ and $b = 15 - 6 = 9$, we get the following solution with formula (8). Then we get

$$X_1^t = (1,1,1,0,0,1,0,0,0,0) \text{ and } f_1^t = 71.$$

Obviously, since $f_1^t > f^t$, we keep in mind the solution X_1 and the number f_1^t for now. Since there are $n_0 = 4$ "0" in the end of the solution X_1, we do not replace other zeros with "1".

Now, if we write $x_5 = 1$ according to solution (19), we will get the following solution with formula (18).

$$X_2^t = (1,1,1,0,1,0,0,0,0,0)$$

The function has the value $f_2^t = 72$ according to this solution.

As we can see, since $f_2^t > f_1^t$, the solution X_2 is a better solution. Therefore, we keep in mind f_2^t and the solution X_2. After that, since $x_5 = \frac{3}{5}$ when $k = 5$ in solution (19), we replace the 4th, 3rd and other coordinates before the 5th coordinate with zero, and construct the next solutions by formula (18).

First note $x_4 = 0$l. By formula (18), we get the solution

$$X_3^t = (1,1,1,0,1,0,0,0,0,0)$$

and accordingly $f_3^t = 72$. Since $f_3^t = f_2^t$, we do not keep it and the solution X_3^t in mind. Now we have to note $x_3 = 0$ in solution (19) and find the next solution using formula (18). However, since $n_1 = 3$, there is no need to perform this step.

Thus, the last remembered solution X_2^t and corresponding f_2^t will be the desired innovative approximate solution and the corresponding innovative approximate value. More precisely,

$$X^{it} = X_2^t = (1,1,1,0,1,0,0,0,0,0)$$

and $f^{it} = f_2^t = 72$.

4. Results of the computational experiment

In order to investigate the quality of the method proposed in the article, an extensive computational experiment was conducted on problems of different dimensions. For this purpose, a program was created based on the algorithm given above. The coefficients of the solved problems were chosen as random numbers, with at most two-digit or at most three-digit natural numbers. So, in type I problems, the coefficients $0 < c_j \leq 99$, $0 < a_j \leq 99$ $\left(j = \overline{1,n}\right)$ $b = \left[\frac{1}{3}\sum_{j=1}^{n} a_j\right]$ were chosen, and in type II problems $0 < c_j \leq 999$, $0 < a_j \leq 999$, $b = \left[\frac{1}{3}\sum_{j=1}^{n} a_j\right]$. Here, the sign [z] indicates the integer part of the number z.

The results are given in the tables below.

Table 1

Problems with two-digit numbers ($n = 100$)

N	1	2	3	4	5
$\overline{f}$	3420.24	3448.41	3737.18	3350.21	3216.77
f^t	3400	3437	3716	3345	3205
f^{it}	3400	3443	3722	3345	3214
l	0	1	2	0	1
δ	0.592	0.331	0. 567	0. 156	0.366
δ^i	0. 592	0. 157	0.406	0. 156	0.086

Table 2

Problems with two-digit numbers ($n = 500$)

N	1	2	3	4	5
$\overline{f}$	16112.89	17103.33	17054.60	16811.72	16982.53
f^t	16109	17096	17047	16805	16979
f^{it}	16112	17101	17053	16811	16979
l	2	2	3	3	0
δ	0.024	0.043	0.045	0.040	0.021
δ^i	0.006	0.014	0.009	0.004	0.021

Table 3

Problems with two-digit numbers ($n = 1000$)

N	1	2	3	4	5
$\overline{f}$	33900.66	33000.13	32955.12	33582.66	33134
f^t	33899	32999	32951	33579	33129
f^{it}	33900	32999	32954	33581	33132
l	1	0	1	3	4
δ	0.005	0.003	0. 012	0.011	0.015
δ^i	0.002	0.003	0.003	0.005	0.006

Table 4

Problems with three-digit numbers ($n = 100$)

N	1	2	3	4	5
$\overline{f}$	34373.71	34605.28	37524.88	33632.46	32364.57
f^t	3424 3	344 90	3733 8	3352 8	3230 6
f^{it}	3424 3	34540	3747 8	3352 8	3230 8
l	0	1	2	0	2
δ	0. 380	0. 333	0. 498	0. 311	0. 181
δ^i	0. 380	0. 197	0. 125	0. 311	0. 175

Table 5

Problems with three-digit numbers ($n = 500$)

N	1	2	3	4	5
$\overline{f}$	162031.83	171859.11	171253.35	168974.04	170664.79
f^t	161962	171797	171239	168895	170627
f^{it}	161996	171835	171239	168920	170627
l	2	2	0	2	0
δ	0.043	0.036	0.008	0.047	0. 022
δ^i	0. 022	0.014	0.008	0.032	0. 022

Table 6

Problems with three-digit numbers ($n = 1000$)

N	1	2	3	4	5
$\overline{f}$	340647.32	331605.19	331255.47	337452.09	332930.90
f^t	340602	331560	331196	337397	332873
f^{it}	340643	331570	331242	337421	332908
l	2	1	3	3	1
δ	0.013	0.014	0.018	0.016	0.017
δ^i	0.001	0.011	0.004	0.009	0.007

The following notation is adopted in the tables:

N – the number of the solved problem.

$\overline{f}$ – the upper bound of the optimal value.

f^t– the approximate value of the objective function.

f^{it}– an innovative improved approximate value of the objective function.

l – number of improvements.

$\delta = \frac{|\overline{f} - f^t|}{|f|} \times 100$ – relative error of the approximate value (in percent).

$\delta^i = \frac{|\overline{f} - f^{it}|}{|\overline{f}|} \times 100$ – the relative error of the innovative approximate value (in percent).

5. Conclusion

Based on the data of the conducted computational experiments given in the above tables, we can draw the following conclusions.

In almost all problems, the approximate solution found by the known method is innovatively improved by the algorithm proposed in this study. Out of 30 solved problems, there was no improvement in only 8. For those problems, the initially found solution may have been the optimal solution.

It can be seen from the tables that the relative error percentages in problems with three-digit numbers are smaller than in problems with two-digit numbers. This is expected.

The relative errors of the upper bound of the optimal value did not exceed 1% with the value given to the objective function of the innovative improved solution proposed in this study. This is important in solving real practical problems.

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