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Optimization of ensuring tactical formation during joint flight

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A B S T R A C T

The procedure of mathematical formulation of tactical formation of UAV groups participating in a joint flight is proposed. An algorithm for controlling the flight of a group along a planned route in a set formation is given, based on the System-of-Systems principles. It is assumed that the control is performed by each UAV autonomously and is based on the information about the positions of neighboring UAVs. The problem of UAV changing formation is also investigated and an optimal solution is proposed.

1. Introduction

The rapid development of information technology, on the one hand, and radio-electronic devices, on the other hand, has led to a variety of applications of unmanned aerial vehicles, including their widespread use in military operations. The use of UAVs in various wars and conflicts around the world in recent years has shown that it is more effective to simultaneously involve a large number of UAVs in operations (field of activity). This has necessitated the study of a number of problems arising from the use of UAVs in military operations.

One of the problems of this kind is setting up possible tactical formations of UAVs flying together with aircraft and managing the transition from one tactical formation to another during operations.

Tactical formation is the arrangement and order of operations of military units in an area of operations for the purpose of successful conduct of combat actions. Tactical formation may vary at different stages of an operation depending on the task at hand, the specifics of the area of operations, the tactical and technical characteristics and armament of aircraft engaged in group flight, and weather conditions.

In military aviation, various tactical formations (column, wedge, etc.) are implemented [1]. In UAVs, they can be similar or different. The aim of this study is to formulate and present the information about the tactical formations UAVs engaged in joint flight have at different stages of the operation.

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For the purpose of this study, we will assume that the UAVs participating in the joint flight are of the same type and their maximum flight speed is v_{max}.

2. Tactical formations in aviation

Currently, there is a lot of information on the Internet resources about various shows with changing the formation of a UAV group [2]. During these types of shows, which are often performed at night, on the one hand, light effects are used (the drones' lights are turned on and off at different times), and on the other hand, the formation in the flight space changes. In the preparation of these shows, an "action" program is prepared for each drone.

UAVs that carry out military operations as a group generally do not have individual tasks, they determine their tasks during the operation and take their place in the tactical formation based on current spatial coordinates.

Unlike spatial drone shows, military aviation tactical formations have a planar arrangement. Among them, the main simple formations are: column, echelon, vanguard, wedge (Fig. 1) and complex formations – echelon of pairs or flights, vanguard of pairs or flights, and wedge of pairs or flights (e.g., Fig. 2).

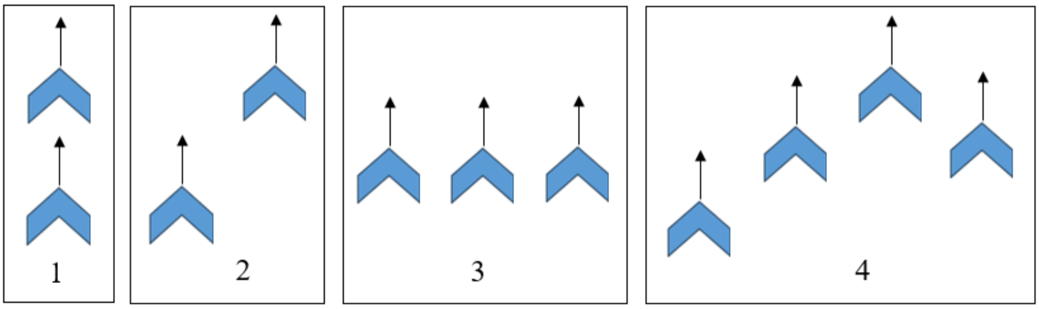

Fig. 1. Simple aviation formations: 1. Column; 2. Echelon; 3. Vanguard; 4. Wedge

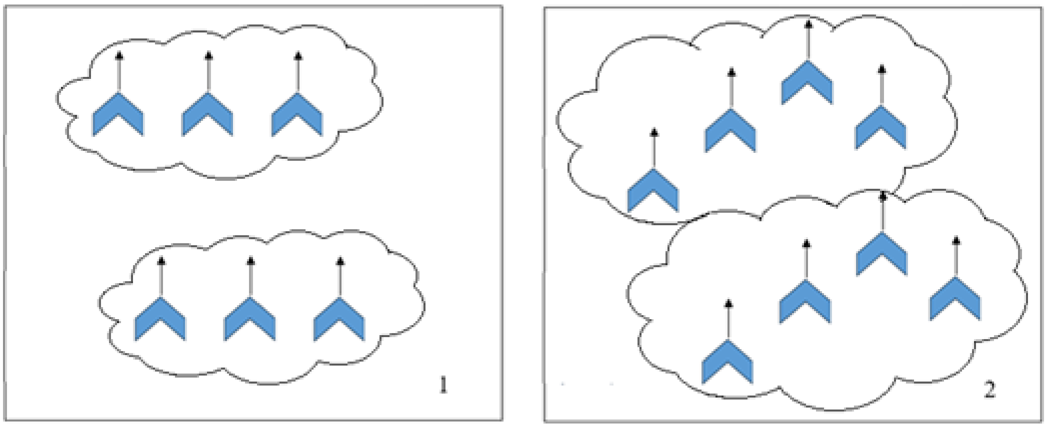

Fig. 2. Complex tactical formations: 1. Column lines; 2. Wedge lines

Without loss of generality, complex tactical formations can be viewed as a sequence of simple tactical formations executed by different groups of UAVs. Therefore, for simplicity, we will consider here the transition of UAVs performing group flight between simplest tactical formations.

3. Formulating tactical formations

The purpose is to ensure that each UAV participating in the joint flight is informed of the intended formation. To this end, introduce the Oxy coordinate system. Since the movement is considered in a plane, the coordinate system will be two-dimensional. The Ox axis will be oriented from left to right, the Oy upward. The coordinate axes are mutually perpendicular. To determine the coordinate origin, let us number the UAVs. The numbering is done in a "lexicographic" order: the topmost UAV is 1st. If there are several points in the first line, they, in turn, are numbered from left to right. In other words, the numbering is done from top to bottom, from left to right. In each current situation under consideration, when studying how the UAVs behave relative to each other for the next period, we will consider the coordinate origin to be the 1st UAV.

To formulate such information, we introduce below the concept of conditional distances A and B between the centers of the UAVs, as shown in Fig. 3.

We will write the information about a simple tactical formation as follows, for the case where there are $N + M$ UAVs in the group:

$$\langle A, B, N, M \rangle.$$

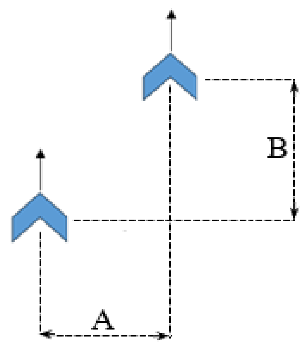

Fig. 3. Procedure for setting the conditional distances A and B.

Here, N is the number of UAVs to the right of the first UAV, including the first UAV, M is the number of UAVs to the left of the first UAV. This can be applied to a case when the number of UAVs differs from $N + M$, then we take that the ratio $N{:}M$ is the "ratio" of the UAVs to the left and to the right of the first UAV. In other words, when the total number of UAVs is not $N + M$, their left to right distribution must be $N{:}M$. If the ratio $N{:}M$ of the total number does not determine an integer, they must be rounded up in a logical manner.

$A = 0$ indicates a trail tactical formation, and $B = 0$ indicates a line tactical formation. From a formation perspective, it does not matter what the values of N and M are in these cases, but for the invariance of the samples, we will take $N = M$ in these cases, for example, $N = M = 1$. The echelon formation differs from the wedge formation in that one of the numbers, N or M, is zero, depending on the flank of the formation, $N = 1$, $M = 0$ or $N = 0$, $M = 1$. In this respect, the echelon formation is a special case of the wedge formation.

Denote the number of UAVs performing joint flights by n. Denote the number of UAVs that are supposed to fly to the right and left of the 1st UAV, including the 1st UAV, in the wedge formation by j_r and j_l. Then:

$$\begin{cases} j_r = \left[\dfrac{nN}{M+N}\right], \\ j_l = n - \left[\dfrac{nN}{M+N}\right], \end{cases} \tag{1}$$

where the square brackets indicate that the integer part of the calculated fraction is taken.

4. Specifics of joint flight to the area of operations

Joint flight of UAVs implies that each of them regularly determines its location (coordinates) based on onboard navigation devices [3]. This information serves, on the one hand, to control its movement along the route, and, on the other hand, to adjust the speed and direction of movement of the UAVs to avoid collisions with each other. Therefore, UAVs participating in a joint flight regularly exchange information, transmitting information about their locations to each other [4]. This allows each UAV to have information about the current coordinates of the others at the time of the information exchange session and to calculate their average speed based on the data from the last two exchange sessions.

It is assumed that the flight route of UAVs in a joint flight to the area of operations is determined in advance and information about obstacles along the route is provided in the format of a "geometric map" [5]. The flight control of each UAV along the route to the area of operations is carried out using the System-of-Systems principles [6]. According to this principle, in order for a UAV to avoid collisions with others, it is enough for it to receive information about drones flying in its immediate vicinity. At this time, the UAVs, naturally, do not follow any tactical formation.

It should be noted that it is not impossible for drones flying in a group to fail for one reason or another. Such a situation can occur, for example, due to technical malfunctions, collisions with unexpected obstacles or other reasons, or interference from external forces. For this reason, the exact number of UAVs arriving at the area of operations may not be known in advance. Therefore, it is necessary to specify the number of UAVs during the “entry” to the area of operations. Below, this number is denoted as n.

5. The mutual state of the UAV group and the tactical formation

The formulation and solution algorithm of the problem of transition to a tactical formation are solved depending on how the initial mutual state of the UAV group and the tactical formation is given. Let us introduce a few concepts related to the mathematical formulation of the problem.

Tactical formation positions. As introduced above, the tactical formation is defined by assigning the numbers $\langle A, B, N, M\rangle$. To determine the location of the positions in the considered formation, let us introduce the formal Oxy coordinate system. For the sake of clarity of the explanation given below, let us orient the coordinate axes in the traditional way according to the image plane, with the Ox axis oriented from left to right and the Oy axis oriented from top to bottom. It is assumed that the UAV group is flying together in the direction of the Oy axis.

Let us look at the wedge tactical formation. Assume that the 1st UAV is at the coordinate origin, the front flank positions of the wedge formation are located in the IV quarter of the Oxy system, and the left flank positions are in its III quarter. We number these positions starting from zero to the right from point O, and then continue the numbering the positions located to the left of point O. Starting from the known total number n of UAVs that have reached the area of operations, it is clear that the number of UAVs on the right and left is calculated by formulas (1). Then we can write the following formulas to calculate the coordinates $\tilde{x}_{0j}, \tilde{y}_{0j}$ of the tactical formation positions relative to the Oxy system:

$$\left(\tilde{x}_{0j}, \tilde{y}_{0j}\right) = \begin{cases} (A \cdot j, -B \cdot j), & j = 0, 1, \dots, j_r - 1, \\ (-A \cdot (j + 1 - j_r), -B \cdot (j + 1 - j_r)), & j = j_r, j_r + 1, \dots, n - 1. \end{cases} \qquad (2)$$

It should be noted that the calculation of the coordinates of tactical order positions corresponding to other tactical formations can also be carried out using formulas (2).

Group center. As mentioned above, it is assumed that the UAVs participating in a joint flight regularly exchange data received from their onboard navigation devices. Based on this data, the location (coordinates) of the UAVs relative to each other in a certain rectangular conditional coordinate system is determined. For simplicity, it can be assumed that all UAVs are at the same altitude. We will number the UAVs that have reached the area of operations as $k = 0, 1, \dots, n - 1$, and denote the coordinates of the k-th UAV in the conditional coordinate system at the instant t by $\left(x_{t,k}, y_{t,k}\right)$. We will call the point whose coordinates are calculated as follows at the instant t the group center corresponding to that instant.

$$\left(x_{*t,k}, y_{*t,k}\right) = \left(\frac{1}{n}\sum_{k=0}^{n-1} x_{t,k}, \frac{1}{n}\sum_{k=0}^{n-1} y_{t,k}\right). \tag{3}$$

Let us assume that the UAV group is moving at a constant speed. Let us take Δt as the time interval such that the constant speed movement continues during that interval. We will assume that during the flight of the UAVs to the operational zone, the group center changes its location along the route, therefore, the ***group movement direction vector*** can be calculated as the difference between the calculated coordinates of the group center at the time $t - \Delta t$ and t, when information is exchanged between the UAVs:

$$\boldsymbol{v}\left(v_x, v_y\right) = \left(\frac{x_{*t,k} - x_{*t-\Delta t,k}}{\Delta t}, \frac{y_{*t,k} - y_{*t-\Delta t,k}}{\Delta t}\right). \tag{4}$$

The ***location of the group*** refers to the points relative to the Oxy coordinate system determined as follows:

$$\begin{cases} L = L\left(\min\limits_{k=0,1,\dots,n-1}\{x_{t,k}\}, \operatorname{mix}\limits_{k=0,1,\dots,n-1}\{y_{t,k}\}\right), \\ R = R\left(\max\limits_{k=0,1,\dots,n-1}\{x_{t,k}\}, \max\limits_{k=0,1,\dots,n-1}\{y_{t,k}\}\right). \end{cases} \tag{5}$$

It is easy to see that the LR segment represents the diameter of the rectangle in which the UAVs performing joint flights are located and whose sides are parallel to the coordinate axes. Thus, if the information $\langle A, B, N, M\rangle$ about the tactical formation and the coordinates of all UAVs that have reached the area of operations at the time $t - \Delta t$ and t are known, the tactical order positions and the location of the group can be calculated using formulas (1)-(5).

To formulate the problem of transition to the wedge tactical formation, let us determine the location of the new tactical order positions relative to the UAV group. Based on the fact that the UAV group moves in the direction of the Oy axis, when determining the positions of the new tactical order, it can be assumed that the 1st UAV belonging to the tactical order should be in the middle of the location of the group. Then its coordinate on the Ox axis is calculated as follows:

$$X(t) = \frac{\min\limits_{k=0,1,\dots,n-1}\{x_{t,k}\} + \max\limits_{k=0,1,\dots,n-1}\{x_{t,k}\}}{2}. \tag{6}$$

On the other hand, since the joint flight is carried out in the direction of the Oy axis, the coordinate of the 1st UAV belonging to the tactical order in order to be ahead in the direction of flight and close to the current location of the UAV group, on the Oy axis, can be determined, for example, as follows:

$$Y(t) = |\boldsymbol{v}|\Delta t + \max\limits_{k=0,1,\dots,n-1}\{y_{t,k}\} + \frac{B}{A}\frac{\max\limits_{k=0,1,\dots,n-1}\{x_{t,k}\} - \min\limits_{k=0,1,\dots,n-1}\{x_{t,k}\}}{2}. \tag{7}$$

Equations (6)-(7) can be satisfied by increasing coordinates (2) to X and Y, respectively. It should be noted that formulas (6)-(7) will be applied during the transition to the line formation.

In the echelon formation, the X coordinate, unlike (6), can be calculated as follows:

$$X(t) = N \cdot \min\limits_{k=0,1,\dots,n-1}\{x_{t,k}\} + M \cdot \max\limits_{k=0,1,\dots,n-1}\{x_{t,k}\}.$$

For the purpose of transitioning to the column formation, the coordinates of the initial mutual position are determined according to the following algorithm:

- First, the numbers $\{x_{t,k}\}$ are arranged in ascending order:

$$\min_{k=0,1,\ldots,,n-1}\{x_{t,k}\} \equiv \xi_0, \xi_1, \xi_2, \ldots, \xi_{n-1} \equiv \max_{k=0,1,\ldots,,n-1}\{x_{t,k}\}.$$

- Two consecutive numbers ξ_i, ξ_{i+1} are taken that are close to $\frac{\xi_0+\xi_{n-1}}{2}$ such that

$$\xi_{i+1} - \xi_i \geq \frac{\xi_{n-1} - \xi_0}{n = 1}$$

If the number of such numbers is more than 1, the $i \to \min$ one is taken.

- Finally, instead of X and Y determined by formulas (6)-(7), the following numbers are taken:

$$X(t) = \frac{\xi_{i+1} - \xi_i}{2},$$

$$Y(t) = |\boldsymbol{v}|\Delta t + \max_{k=0,1,\ldots,,n-1}\{y_{t,k}\}.$$

6. The problem of determining the location of UAVs in the tactical formation

This problem involves automating the process of UAVs changing positions relative to each other so that the UAVs that have flown along the route and reached the area of operations take the intended tactical formation. It is clear that the UAVs that perform group flights actually fly irregularly until they reach the area of operations, provided that they do not collide with each other. Therefore, the relative positions of the UAVs that have reached the area of operations, as a rule, do not correspond to the intended tactical formation positions. In order to implement the transition of the UAVs to the tactical formation, it is necessary to determine which j-th position of each k-th UAV should move to. It is clear that such transition rules can be of various types, and it is normally required to determine the optimal one from the possible transition rules.

The optimality criterion can be taken as the minimum of the total energy required for the UAVs participating in the joint flight to take t the tactical order positions. Considering that the energy expended on the path traveled is proportional to the length of the path, the condition of minimum energy is established as the condition of minimum of the total length of the paths traveled.

At the considered instant, the position of the k-th UAV is considered known. In a given time interval, this UAV must move to the j-th position in the new formation. The position of the j-th UAV in the new formation is determined depending on relations (6), (7) and the type of formation. Let us denote the distance between the current k-th and new j-th positions as d_{jk}. It is clear that the quantities d_{jk} are non-negative numbers.

To solve the problem, let us introduce the number z_{jk}, which takes only the values "0" and "1": $z_{jk} = 1$ will indicate that the k-th UAV is directed to the j-th position during the transition to the tactical formation, while $z_{jk} = 0$ will indicate that it is not.

It is clear that there can only be one non-zero number in the series of numbers $\{z_{jk}, j = 0, 1, \ldots, n-1\}$, so the same UAV can occupy only one position in the tactical formation:

$$\sum_{j=0}^{n-1} z_{jk} = 1, \qquad k = 0, 1, \ldots, n-1. \tag{8}$$

Also, the condition that there can only be one UAV in each position is written as follows:

$$\sum_{k=0}^{n-1} z_{jk} = 1, \qquad j = 0, 1, \dots, n-1. \tag{9}$$

According to the above criterion, the optimal transition to the battle formation involves finding only the numbers $\{z_{jk}, j = 0, 1, \dots, n-1,\ k = 0, 1, \dots, n-1\}$ that take the values "0" and "1" such that the following sum takes the minimum value:

$$J \equiv \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} d_{jk} z_{jk}. \tag{10}$$

From a mathematical point of view, problem (8)-(10) is an assignment problem, and its solution methods (e.g., the Hungarian method) are well known [7, p.142], [8, p.38]. The solution of problem (8)-(10) determines the position of each of the UAVs participating in the joint flight in the tactical formation.

7. Solving the problem of implementing a tactical formation

It should be noted that during the implementation of the tactical formation of UAVs participating in the group flight, it must be ensured that they fly without collisions based on the solution of problem (8)-(10). The mutual position of the UAV group and the tactical formation is determined by formulas (6)-(7) above in such a way that none of the UAVs gets in the straight lines defining the geometric location of the tactical order positions. We will show that in this case the following statement is true:

Theorem. For a solution to problem (8)-(10) that satisfies the condition

$$J \to \min \tag{11}$$

the trajectories (straight line segments) connecting the UAVs and their assigned positions in the layout do not intersect.

To prove this proposition, let us assume the opposite. Suppose that this is not the case, but that there are at least two UAVs whose rectilinear flight paths to their destinations intersect at a certain point. Suppose that the numbers of these UAVs and their destinations are k_1, k_2 and j_1, j_2-, respectively. This solution is implemented through $z_{j_1k_1} = 1$ and $z_{j_2k_2} = 1$ in the series $z_{jk} = 1$.

Let us denote the geometric locus of the positions of those UAVs as A_1, A_2 and B_1, B_2, and let the intersection point of the trajectories A_1B_1 and A_2B_2 be C (Fig. 4).

According to the condition of the problem, minimum (11) is provided by the distances $d_{j_1k_1} = |A_1B_1|$ and $d_{j_2k_2} = |A_2B_2|$. According to the triangle equality

$$|A_1B_2| < |A_1C| + |CB_1| = |A_1B_1|,$$

$$|A_2B_1| < |A_2C| + |CB_2| = |A_2B_2|.$$

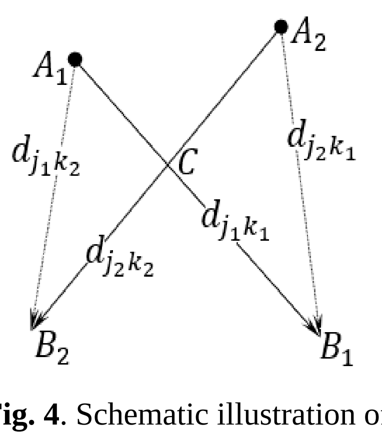

Fig. 4. Schematic illustration of two UAVs with intersecting trajectories and corresponding destinations.

Therefore,

$$d_{j_1k_2} + d_{j_2k_1} \equiv |A_1B_2| + |A_2B_1| < |A_1B_1| + |A_2B_2| = d_{j_1k_1} + d_{j_2k_2}.$$

Taking into account this inequality,

$$J = \left(J - (d_{j_1k_1} + d_{j_2k_2})\right) + \left(d_{j_1k_1} + d_{j_2k_2}\right) > \left(J - (d_{j_1k_1} + d_{j_2k_2})\right) + \left(d_{j_1k_2} + d_{j_2k_1}\right) = J'.$$

The value J' is implemented by replacing the values$z_{j_1k_1} = 1$, $z_{j_2k_2} = 1$, $z_{j_1k_2} = 0$, $z_{j_2k_1} = 0$ in the set $\{z_{jk}\}$ with the values $z_{j_1k_1} = 0$, $z_{j_2k_2} = 0$, $z_{j_1k_2} = 1$, $z_{j_2k_1} = 1$. It turns out that there is a possible solution to the problem (8)-(10) that realizes a smaller value of the corresponding functional (11). This contradicts the condition of minimality of J.

Thus, the counter-hypothesis that the trajectories connecting the UAVs and positions intersect is not true. The proposition is proven.

The validity of the proposition shows that the trajectories implementing the tactical formation determined on the basis of the solution of problem (8)-(10) do not intersect, therefore, the UAVs can occupy the tactical formation positions without the danger of colliding with each other.

Assume that the tactical formation positions are expected to be occupied during the period δt.

Suppose

$$V = \max\left\{\sqrt{\left(\frac{\tilde{x}_{\gamma j} - x_{t,k}}{\delta t}\right)^2 + \left(\frac{\tilde{y}_{\gamma j} - y_{t,k}}{\delta t}\right)^2}\right\}$$

Denote

$$\begin{cases} \lambda_1 = \dfrac{\aleph \cdot v_{max}}{V}, \\ \lambda_2 = \dfrac{(1 - \aleph) \cdot v_{max}}{V} \end{cases}$$

where $\aleph$ is the speed distribution coefficient in directions, $\aleph < 1$. In numerical calculations, we can take $\aleph = 0.1$. In order for the k-th UAV to occupy the combat formation position at the instant δt, its velocity should be adjusted as follows

$$\boldsymbol{v_k}\left(v_{x,k}, v_{y,k}\right) = \left(\lambda_1 v_x + \lambda_2 \frac{\tilde{x}_{\gamma j} - x_{t,k}}{\delta t}, \lambda_1 v_y + \lambda_2 \frac{\tilde{y}_{\gamma j} - y_{t,k}}{\delta t}\right).$$

It should be noted that after the process of transitioning to the tactical formation is completed, all UAVs are expected to continue their movement at the speed $\boldsymbol{v}\left(v_x, v_y\right)$.

8. Conclusion

The procedure of mathematical formulation for each formation during the joint flight of a group of UAVs is proposed. An algorithm for controlling the flight of a group of objects along a given trajectory in a given formation is given. The algorithm of transition to one of tactical formations after occupying a combat position is proposed. The control is performed by each UAV autonomously and is based on information about the positions of neighboring UAVs. An optimal solution for transition from one formation to another is proposed.

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