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# Optimization of source parameters in transport networks

**Yegana Ashrafova**

*Baku State University, Baku, Azerbaijan*
*Institute of Control Systems, Baku, Azerbaijan*

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**ABSTRACT**

*This study centers on optimizing parameters related to concentrated sources that impact the functioning of transport networks. The transport network is composed of multiple one-dimensional objects, each following by a system of ordinary differential equations with nonlocal boundary conditions to characterize their states. The sources exert influence on specific points within these objects and their connection points. The goal of the optimization process is to identify the best locations and power levels for these sources, guided by predefined criteria for target functional. We establish the necessary optimality conditions for the parameters of concentrated sources.*

## 1. Introduction

The optimization problem, which encompasses the determination of the placement and power allocation of concentrated sources [1-8] that affect the efficiency of a complex dynamic system, has been thoroughly examined. This intricate transport system consists of numerous one-dimensional elements, each of which is governed by a system of ordinary differential equations with nonlocal boundary conditions. The interconnection of these elements occurs in arbitrary sequences, relying exclusively on the states at their respective starting and ending points.

We designate intricate transport networks as dynamic objects with a network structure [8-10], and the corresponding mathematical models are termed network models. These objects are conveniently visualized as directed graphs. It is crucial to emphasize that these graphs are typically incomplete, with most entries in the connection matrix being zero. Nonzero entries in this matrix signify connections between the initial and final states of individual blocks corresponding to adjacent links in the graph.

The focal problem involves numerous challenges, particularly the optimal control of transient processes in the dynamic flow of liquids or gases within complex pipeline transport networks [8,9]. The mathematical models governing these processes consist of subsystems of hyperbolic partial differential equations, each describing fluid movement within a distinct section of the pipeline. At the junctions between these sections, continuity of flow and material balance conditions must be maintained, resulting in nonlocal boundary conditions. The application of the method of lines in either time or spatial variables (analogous to the decomposition method) transforms the problem of controlling material flow within the transportation network into the issue addressed in this article.

*E-mail address*: ashrafova.yegana@gmail.com (Yegana Ashrafova)

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## 2. Problem statement

We examine a complex transport network comprising m links (blocks) that are interconnected at their ends in a random manner. The structure of this entity is conveniently depicted as a directed graph.

The set of all vertices of the graph is represented as $V$, and the set of links $(i,j)$ of length $l^{i,j}$ with a beginning at the vertex $i \in V$ and an end at the vertex $j \in V$ is denoted by $E = \{(i,j): i,j \in V\}$, $|V| = N$, $|E| = K$, $|V|$ signifies the number of elements in set $V$.

Let the sets of links $E_i^+ = \{(j,i): j \in V_i^+\}$, $E_i^- = \{(i,j): j \in V_i^-\} -$ respectively entering and leaving the $i$-th vertex, $V_i^+$ and $V_i^- -$ the sets of vertices adjacent to the $i$-th vertex, which are respectively the ends and beginnings of links from the set $E_i$, $E_i = E_i^+ \cup E_i^-$, $V_i = V_i^+ \cup V_i^-$. Let us denote

$$|E_i^+| = |V_i^+| = \bar{n}_i, \qquad |E_i^-| = |V_i^-| = \underline{n}_i, \ \bar{n}_i + \underline{n}_i = n_i, \qquad i \in V .$$

It's clear that

$$\sum_{i \in V} \underline{n}_i + \sum_{i \in V} \bar{n}_i = K, \ \sum_{i \in V} n_i = 2K .$$

In practical applications, as a rule, the relation $n_i \ll N$, $i \in V$, holds, i.e. the number of vertices adjacent to any vertex is much less than the total number of vertices.

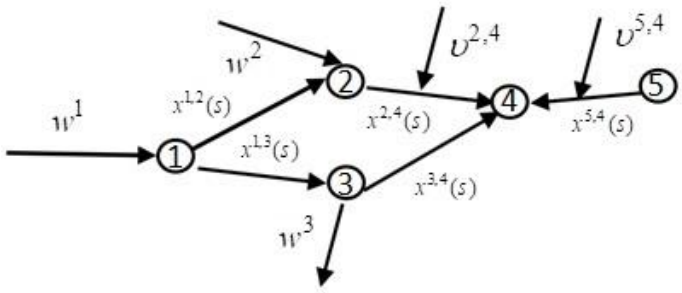



**Fig. 1.** Scheme of transport network

Each link in the graph is associated with an independent subobject (block). Let the state of each of the links $(i,j) \in E$, $j \in V_i^-$, $i \in V\}$ be described by a system of $m$ linear nonautonomous ordinary differential equations (Fig.1)

$$\frac{dx^{ij}(s)}{ds} = A^{ij}(s)x^{ij}(s) + B^{ij}\upsilon^{ij}\delta\left(s - \xi^{ij}\right) + f^{ij}(s), \qquad s \in \left(0, l^{ij}\right) \tag{1}$$

with $M_i$, $M_i \le n_i \cdot m$, linearly independent boundary conditions specified in undivided form

$$\sum_{j \in V_i^-} g_t^{ij} x^{ij}(0) + \sum_{j \in V_i^+} q_t^{ji} x^{ji}\left(l^{ji}\right) = w_t^i, \quad \mathrm{t} = \overline{1, M_i}, \quad i \in V. \tag{2}$$

Here functions $x^{ij}(s) = x^{ij}(s;u) \in \mathrm{R}^m$ characterizes the state of the $(i,j)$ -th link with length $l^{ij}$ at the point $s \in \left[0, l^{ij}\right]$; $u = (\upsilon, w, \xi) -$ vector of parameters to be optimized, whose parameters are $\upsilon \in \mathrm{R}^{\mu}, \upsilon = (\upsilon^{ij} = (\upsilon_1^{ij}, \ldots, \upsilon_{\mu_{ij}}^{ij}) \in \Omega_{\upsilon^{ij}} \subset \mathrm{R}^{\mu_{ij}}: j \in V_i^-, i \in V)$, $\upsilon_t^{ij} - t$ - th component $\mu_{ij}$ - dimensional $(i,j)$ - th external source acting on the $(i,j)$ th subsystem at the point $\xi^{ki} \in \Omega_{\xi^{ki}} \equiv \left[0, l^{ij}\right]$, $\xi = (\xi^{ij}: j \in V_i^-, i \in V)$, $\xi \in \mathrm{R}^{\mathrm{k}}, \mathrm{k} \le K$; $\mathrm{w} \in \mathrm{R}^M$, $\mathrm{w} = \left(\mathrm{w}^i \in \Omega_{\mathrm{w}^i} \subset \mathrm{R}^{M_i}, i \in I\right)$, $\mathrm{v}^i = \left(\mathrm{w}_1^i, \ldots, \mathrm{w}_{M_i}^i\right)^{\mathrm{T}}, w_t^i - t$ - is the component of the external source acting on the $i$ th vertex. Let's denote

$$\mu = \sum_{i\in V}\sum_{j\in V_j^-}\mu_{ij}\,,\ \ M = \sum_{i=1}^{N} M_i.$$

The given in the problem are: $A^{ij}(s) \neq \text{const}$, $f^{ij}(s)$ −respectively $m$- dimensional square matrix and vector continuous at $s \in [0, l^{ij}]$ functions; $B^{ij} - (m \times \mu_{ij})$- dimensional scalar matrices; row vectors $g_t^{ij} = \left(g_{t,1}^{ij}, \ldots, g_{t,m}^{ij}\right)$, j$\in V_i^-$, $q_t^{ji} = \left(q_{t1}^{ji}, \ldots, q_{tm}^{ji}\right), j \in V_i^+$, $t = \overline{1, M_i}, i \in V$. If $B^{ij} = 0_{m\times\mu_{ij}}$, then this means that there are no sources in the $(i,j)$-th section.

In practical problems, depending on the sign of the source parameters $v^{ij}, j \in V_i^-,\ i \in V$, the external source at a point $\xi^{ij}$ is called "outflow" or "inflow".

The total number of subsystems in equation (1) is equal to the number of links $K$ in the network, each of which connects to adjacent links (blocks) in an arbitrary order solely through non-separated (nonlocal) boundary conditions [11], as described in equation (2). The total number of differential equations in system (1) is equal to $Km$, and the number of boundary conditions in equation (2) should also be equal to $M$ ensuring compatibility between equations: $M = Km$.

We will assume that the boundary value problem (1), (2) has a unique solution. This, as is known, depends only on matrices $A^{ij}(s),\ j \in V_i^-$, vectors $g_t^{ij}, j \in V_i^-, q_t^{ji}, \in V_i^+, t = \overline{1, M_i}, i \in V$, and does not depend on other data involved in the problem, in particular, on unknown vectors $v, w, \xi$.

Based on practical considerations, restrictions are imposed on the values of the parameters $v^{ij},\ \xi^{ij},\ \mathrm{w}^i, j \in V_i^-,\ i \in V$, optimized in the problem:

$$v^{ij} \in U_{v^{ij}},\ \ \mathrm{w}^i \in U_{w^i},\ \ \xi^{ij} \in U_{\xi^{ij}} \equiv \left[0; l^{ij}\right],\ \ U^{ij} = U_{v^{ij}} \times U_{w^i} \times U_{\xi^{ij}}. \tag{3}$$

We will assume that the sets of admissible values $U_{v^{ij}},\ U_{w^i}$ are convex and compact. It is required to find such values of the vector components $u = (v, w, \xi)$ for which the functional

$$J(u) = \sum_{i\in V}\sum_{j\in V_i^-}\int_0^{l^{ij}} \left\|x^{ij}(s;u) - X^{ij}(s)\right\|_{R^m}^2 ds + \mathbb{R}(u) \tag{4}$$

$$\mathbb{R}(u) = \sum_{i\in V}\left[\sum_{j\in V_i^-}\left\{\varepsilon_1\left\|v^{ij} - \hat{v}^{ij}\right\|_{R^{\mu_{ij}}}^2 + \varepsilon_2\left\|\xi^{ij} - \hat{\xi}^{ij}\right\|_{R^{\nu_{ij}\times m}}^2\right\} + \varepsilon_3\left\|\mathrm{w}^i - \hat{w}^i\right\|_{R^{M_i}}^2\right]$$

gets the minimum value.

The optimized finite-dimensional vector $u = (v, w, \xi)$, which determines the parameters and locations of external sources, in real problems has a small dimension, despite the large dimension of the system of differential equations (1) itself.

The formulated problem (1) - (4) can be attributed to the class of parametric optimal control problems. The optimized finite-dimensional vector $u$ determining the parameters and locations of external sources in real problems has a small dimension, despite the large dimension of the system of differential equations (1) itself. The solution of the boundary value problem (1), (2) for given values of the vector $u$ has a certain computational complexity. The complexity is due to unseparated (nonlocal) boundary conditions and, of course, the large dimension of the system of differential equations (1) itself. It is necessary to repeatedly determine the value of the functional for the current values of the optimized vector $u$, therefore, to solve the boundary value problem (1), (2), while solving the optimization problem numerically. Therefore, it is important to use both effective methods for solving the optimization problem, so pay special attention to solving the boundary value problem (1), (2).

## 3. Formulas for the solution to the problem

The convexity and differentiability of the functional (4) are studied, formulas for the gradient of the functional with respect to the parameters to be optimized are derived.

Theorem 1. Let all the conditions imposed on the functions and parameters involved in the problem (1) -(4) be satisfied. The functional $J(v,w,\xi)$ is convex in $v,w$, for a fixed admissible vector $\xi$.

It is easy to prove that the functional $J(u)=J(v,w,\xi)$ is not convex on $\xi$ if the condition $B^{ij}v^{ij}\neq 0$ is satisfied for at least one section $(i,j)\in E$, i.e. there are links that are influenced by external sources.

The differentiability of the functional (4) and the formulas obtained in the work for the components of its gradient over the optimized triple $u=(v,w,\xi)$ are investigated.

In many practical applications, external sources do not participate at all links and vertices of the object. In particular, on some links or vertices their values may be set and not optimized. In these cases, the corresponding components of the gradients of the functional $\mathrm{grad}_v J(u)$, $\mathrm{grad}_w J(u)$, $\mathrm{grad}_\xi J(u)$ are not calculated and are taken equal to zero.

To determine the optimal values of $u=(v,\mathrm{w},\xi)$, using formulas for calculating the gradient components of the functional of the problem (1)-(4), one can use effective first-order optimization methods, for example, the gradient projection method [12].

We introduce the following notation. Denote by, $\underline{c}_t^i=\left(g_t^{i1},\dots,g_t^{i\underline{n}_i}\right)$, $\bar{c}_t^i=\left(q_j^{1i},\dots,q_j^{\bar{n}_i i}\right)$ −the row vectors of dimension $\underline{n}_i\cdot m$ and $\bar{n}_i\cdot m$ respectively, $x^i=\left(\underline{x}^i,\overline{x}^i\right)^T$ −extended vector of dimension $n_i\cdot m$, where $\underline{x}^i=(x^{i1}(0),\dots,x^{i\underline{n}_i}(0))^T$, $\overline{x}^i=(x^{1i}(l^{1i}),\dots,x^{\bar{n}_i i}(l^{\bar{n}_i i}))^T$. Let $C_i=\left(c_{ts}^i\right)_{t=1,s=1}^{M_i,n_i\cdot m}$, $i\in V$ is an extended matrix, each row of which is extended row vector $c_j^i=(\underline{c}_j^i,\bar{c}_j^i)$ of dimension $n_i\cdot m$.

According to the assumption of linearly independence of conditions (2) takes place

$$rang C_i=M_i. \tag{5}$$

Since the matrix $C_i$ is of dimension $M_i\times(n_i\cdot m)$, $M_i\le n_i\cdot m$, $i\in V$, then from the matrix $C_i$ we can extract the invertible submatrix (minor) $\hat{C}_i$ with a rank equal to $M_i$. By changing the order of the columns, we again denote the extended matrix by $C_i=[\hat{C}_i,\check{C}_i]$. Here $\check{C}_i$ is the matrix, composed from columns of an extended matrix $C_i$, not included in the matrix $\hat{C}_i$. Similarly, corresponding to the matrix $\hat{C}_i$, the vector $x^i$ is divided into the vector $\hat{x}^i=(\hat{x}_1^i,\dots,\hat{x}_{M_i}^i)^T$ of dimension $M_i$ and the vector $\check{x}^i=(\check{x}_1^i,\dots\check{x}_{(n_i\cdot m)-M_i}^i)^T$ of dimension $(n_i\cdot m-M_i)$. Let $\hat{p}_t$, $t=1,..,M_i$ −are the numbers of columns of the matrix $C_i$, included in the matrix $\hat{C}_i$, and $\check{p}_t$, $t=1,..,(n_i\cdot m-M_i)$ − are the numbers of columns of the matrix $C_i$, included in the matrix $\check{C}_i$.

Let select vectors $\hat{\psi}^i=\left(\hat{\psi}_1^i,\dots,\hat{\psi}_{M_i}^i\right)^T$ of dimension $n_i\cdot m$. from extended vectors, $\psi^i=\left(\left(-\underline{\psi}^i\right)^T,\left(\bar{\psi}^i\right)^T\right)^T$, $\underline{\psi}^i=(\psi^{i1}(0),\dots,\psi^{i\underline{n}_i}(0))^T$, $\overline{\psi}^i=(\psi^{1i}(l^{1i}),\dots,\psi^{\bar{n}_i i}(l^{\bar{n}_i i}))^T$ with component numbers corresponding to column numbers from $C_i$ included in the matrix $\hat{C}_i$.

The following theorem takes place.

Theorem 2. Suppose all the conditions imposed on the functions and parameters in problem (1)–(4) are satisfied. Then the functional (4) is differentiable and components of the gradient of the functional (4) with respect to optimized parameters $v^{ij},\ \xi^{ij},\ w^i, j \in V_i^-,\ \ i \in V$, are determined by the formulas

$$\nabla_{v^{ij}} J(u) = -\left(B^{ij}\right)^T \psi^{ij}\left(\xi^{ij}\right) + 2\varepsilon_1\left(v^{ij} - \tilde{v}^{ij}\right), \tag{6}$$

$$\nabla_{\xi^{ij}} J(u) = -\left(B^{ij}\right)^T \left.\frac{d\psi^{ij}(s)}{ds}\right|_{s=\xi^{ij}} v^{ij} + 2\varepsilon_2\left(\xi^{ij} - \tilde{\xi}^{ij}\right), \tag{7}$$

$$\nabla_{w^i} J(u) = \left(\hat{C}_i^{-1}\right)^T \hat{\psi}^{\,i} + 2\varepsilon_3\left(w^i - \hat{w}^i\right), \tag{8}$$

where continuously differentiable vector functions $\psi^{ij}(s) \in \mathrm{R}^m, s \in [0, l^{ij}], j \in V_i^-,\ \ i \in V$ are the solutions to the adjoint system of differential equations:

$$\frac{d\psi^{ij}(s)}{dx} = -\left(A^{ij}(s)\right)^T \psi^{ij}(s) + 2\left[x^{ij}(s,u) - X^{ij}(s)\right], \qquad s \in \left[0, l^{ij}\right],\ \ j \in V_i^-,\ \ i \in V \tag{9}$$

with unseparated boundary conditions:

$$\check{\psi}^i - \left(\check{C}_i\right)^T \left(\hat{C}_i^{-1}\right)^T \hat{\psi}^{\,i} = 0, \tag{10}$$

Here $\hat{\psi}^{\,i} = \left(\psi^i_{\hat{p}_1}, \ldots, \psi^i_{\hat{p}_{M_i}}\right)^T, \check{\psi}^i = (\psi^i_{\check{p}_1}, \ldots \psi^i_{\check{p}_{(n_i \cdot m) - M_i}})^T$.

It is interesting to note the following. First, the adjoint problem (9), (10) has the same specificity as the direct problem. Namely, the system (9) has a block structure, and the boundary conditions (10) at all vertices are unseparated. Second, in the expressions of the components of the functional gradient with respect to the parameters of the $(k, i)$ -th subsystem, the adjoint function of only this subsystem is involved, as can be seen from (6), (7). In addition, in the formulas for the components of the functional gradient with respect to the sources influencing the $i$-th vertex, as can be seen from (8), the boundary values of the direct and adjoint variables defined just at this vertex are involved.

## 4. Results of numerical experiments

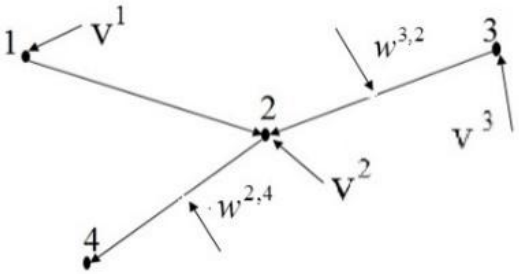



**Fig. 2.** Scheme of object

Let us present the results of numerical experiments obtained when solving the problem of optimizing the values of parameters and locations of sources for an object (Fig. 2), in which

$V = \{1,2,3,4\},\ E = \{(1,2),(3,2),(2,4)\},\ N = 4,\ K = 3, m = 2, M = 6,\ V_1^+ = \emptyset, V_1^- = \{2\},$ $V_2^+ = \{1,3\}, V_2^- = \{4\}, V_3^+ = \emptyset, V_3^- = \{2\}, V_4^+ = \{2\}, V_2^- = \emptyset, l^{i,j} = 1, (j,i): j \in V_i^+, i \in V.$

$$A^{1,2}(s) = \begin{pmatrix} 0 & 1 \\ x & -2 \end{pmatrix}, A^{3,2}(s) = \begin{pmatrix} 0 & 1 \\ 1 & -x \end{pmatrix},$$

$$A^{2,4}(s) = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}, B^{1,2}(s) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$B^{3,2}(s) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad B^{2,4}(s) = \begin{pmatrix} -1 \\ 0 \end{pmatrix},$$

$$f^{1,2}(s) = \begin{pmatrix} s^2 - 3s - 2 \\ 5s + 3 \end{pmatrix},$$

$$f^{3,2}(s) = \begin{pmatrix} 2 - 4s^2 + s \\ 5 + 3s^2 - 2s \end{pmatrix}, f^{2,4}(s) = \begin{pmatrix} -s - 2 \\ 5 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} g_{1,1}^{1,2} & g_{1,2}^{1,2} \end{pmatrix} = (1 \quad 0),\ C_3 = \begin{pmatrix} g_{1,1}^{2,3} & g_{1,2}^{2,3} \end{pmatrix} = (1 \quad 0),\ C_4 = \begin{pmatrix} g_{1,1}^{4,2} & g_{1,2}^{4,2} \end{pmatrix} = (1 \quad 0),$$

$$C_2 = \begin{pmatrix} g_{1,1}^{2,4} & g_{1,2}^{2,4} & q_{1,1}^{1,2} & q_{1,2}^{1,2} & q_{1,1}^{3,2} & q_{1,2}^{3,2} \\ g_{2,1}^{2,4} & g_{2,2}^{2,4} & q_{2,1}^{1,2} & q_{2,2}^{1,2} & q_{2,1}^{3,2} & q_{2,2}^{3,2} \\ g_{3,1}^{2,4} & g_{3,2}^{2,4} & q_{3,1}^{1,2} & q_{3,2}^{1,2} & q_{3,1}^{3,2} & q_{3,2}^{3,2} \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix},$$

Functional is in the following form:

$$J(u) = \int_0^{l^{1,2}} [x_2^{1,2}(s,u) - X_2^{1,2}(s)]^2 ds + \int_0^{l^{3,2}} [x_2^{3,2}(s,u) - X_2^{3,2}(s)]^2 ds + \int_0^{l^{2,4}} [x_2^{2,4}(s,u) - X_2^{2,4}(s)]^2 ds,$$

To construct the model problem, it was assumed that the functions that determine the desired values of the first components of the state of the subobjects are specified and their graphs are shown in Fig. 2. These functions were obtained by numerically solving the boundary value problem for the following parameters

$$w^{1*} = (1), w^{2*} = (0; 0; 1), w^{3*} = (-2), v^{3,2} = (0,2), v^{2,4} = (0,3), \xi^{3,2} = 0.5\,, \xi^{2,4} = 0.6.$$

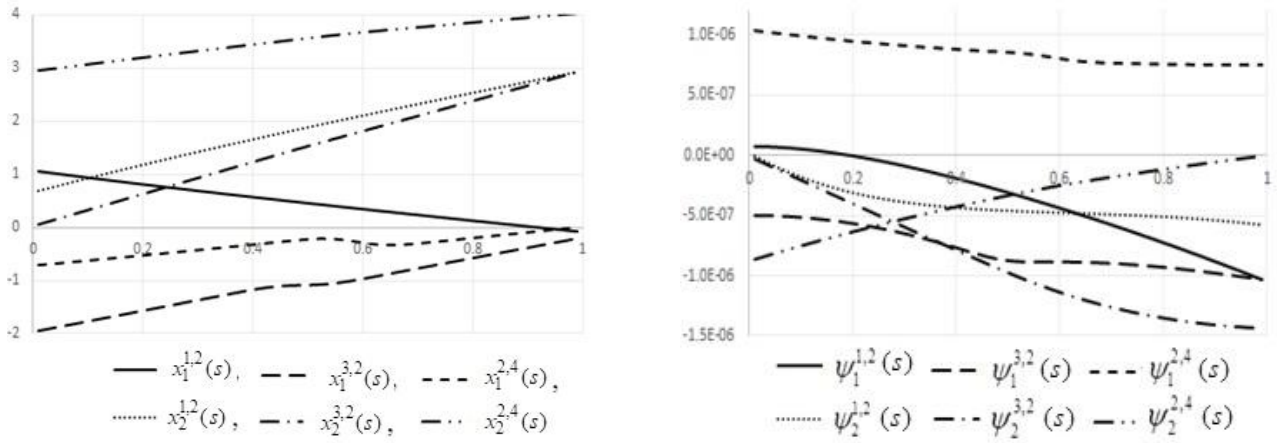
The optimization problem was solved for different initial vector values using the gradient projection method. To solve the direct and conjugate boundary value problems, we used the sweep method based on work [11] and obtained at the result Cauchy problems were solved by the fourth order Runge–Kutta method with steps h=0.01. To approximate the Dirac -function, the Gaussian function was used.

The table shows the values of the optimal parameters $u^*$ obtained by the gradient projection method for four different initial parameter values $u^0$.

**Table 1.**

Results of numerical solution of the problem

| | $u^0 = (v^0, w^0, \xi^0)$ | $u^* = (v^*, w^*, \xi^*)$ | $J(u^0)$ | $J(u^*)$ |
|---|---|---|---|---|
| 1 | ((0,2; 0,72);<br>(-1,7; (0; 1; -2,8); 0);<br>(0,2; 0,36)) | (0,1998; 0,2999);<br>(0,9999; (0,0001; 0; -1); -1,999);<br>(0,5; 0,5999) | 0,7713 | $2 \cdot 10^{-9}$ |
| 1 | ((0,3; 0,8);<br>(-0,7; (0; 1,7;-2,7); 0,7);<br>(0,43; 0,5)) | ((0,2033; 0,2960);<br>(1,0001; -0,0038; -0,0002; -0,997); -2,0011);<br>(0,5001;0,6001)) | 1,2894 | $1.4 \cdot 10^{-6}$ |
| 2 | ((0,6; 0,12);<br>( -1; (0; 1,7; -2,7); 0,7);<br>( 0,43; 0,5)); | ((0,2051; 0,3362);<br>(0,9928; (-0,001; 0,0063; -1,008); -2,0013);<br>(0,49943; 0,5957) | 1,5060 | 0,0001 |
| 3 | ((0,28; 0,2);<br>( -1; (0; 1,7; 0,7); -2,7);<br>( 0,43 ; 0,45 )); | ((0,2002; 0,3005);<br>(1,0002; (-0,0016; -0,0001; -1) -1,9999);<br>(0,4998; 0,5999)) | 1,1639 | $4.4 \cdot 10^{-8}$ |
| 4 | ((0,28; 0,2);<br>( -1,8; (0; 1,7; 0); -2);<br>( 0,3; 0,6 )) | ((0,20004; 0,2999);<br>(1; (0; 0,0002; -0,9999) -2);<br>(0,5; 0,59999)) | 0,2221 | $6.4 \cdot 10^{-10}$ |




**a)** **b)**

**Fig. 3.** Graphs of functions obtained as a result of solving the problem

In Fig. 3,a and 3,b show graphs, respectively, of the solution of the direct and conjugate problems for the obtained parameter values $u^* = (v^*, w^*, \xi^*)$ using the gradient projection method.

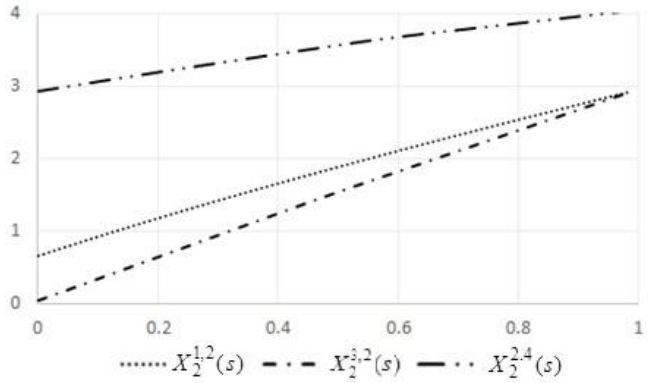



**Fig. 4.** Graphs of desired functions

## 5. Conclusion

This study introduces an optimization problem in transport networks characterized by a system of large dimensional ordinary differential equations. The equations exhibit a block structure, with the blocks interconnected in a nonsequential manner solely through non-separated initial or final values of the phase variables. The investigation in this work focuses on the convexity and differentiability of the functional, and the derivation of necessary optimality conditions concerning both the parameters and concentration locations of sources.

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